UNIVERSITY OF KERALA

DRAFT

Syllabus for

M.Sc. Programme in Mathematics [PG]

with effect from 2020 Admission

SUBMITTED BY BOARD OF STUDIES IN MATHEMATICS [PG]

KERALA UNIVERSITY SYLLABUS FOR M.Sc. MATHEMATICS SEMESTER PATTERN IN AFFILIATED COLLEGES 2020 ADMISSION ONWARDS

M.Sc. MATHEMATICS COURSE STRUCTURE & MARK DISTRIBUTION

Semester		Title of the paper	Distribution hrs per semester	Instructional hrs./ week		Dur ESA hrs.	Maximum Marks		
I				L	P		CA	ESA	Total
	MM 211	Linear Algebra	108	6	-	3 hrs	25	75	100
	MM 212	Real Analysis - I	108	6	-	“	25	75	100
	MM 213	Ordinary Differential Equations	108	6	-	“	25	75	100
	MM 214	Topology - I	126	7	-	“	25	75	100
II	MM 221	Abstract Algebra	108	6	-	3 hrs	25	75	100
	MM 222	Real Analysis-II	108	6	-	“	25	75	100
	MM 223	Topology-II	126	7	-	“	25	75	100
	MM 224	Partial differential Equations and Calculus of Variation	108	6		“	25	75	100
III	MM 231	Complex Analysis	126	7	-	3 hrs	25	75	100
	MM 232	Functional Analysis-I	108	6	-	“	25	75	100
	MM 233	Elective-I	108	6	-	“	25	75	100
	MM 234	Elective-II	108	6	-	“	25	75	100
IV	MM 241	Analytic Number Theory	126	7	-	3 hrs	25	75	100
	MM 242	Functional Analysis-II	108	6	-	“	25	75	100
	MM 243	Elective-III	108	6	-	“	25	75	100
	MM 244	Elective-IV	108	6	-	“	25	75	100
	MM 245	Dissertation/ Project						80+20	100
		Comprehensive Viva							100
		GRAND TOTAL	**1800**						**1800**
L: Lecture; P: Practical; CA: Continuous Assessment; ESA: End Semester Examination									

M.Sc *MATHEMATICS*

(Revised Syllabus from 2020 Admissions)

LIST OF COURSES

SEMESTER– I

MM211 Linear Algebra (Previous Syllabus)
MM212 Real Analysis - I (Revised Syllabus)
MM213 Ordinary Differential Equations (New Syllabus)
MM214 Topology – I (Previous Syllabus)

SEMESTER – II

MM221 Abstract Algebra (New Syllabus)
MM222 Real Analysis - II(Revised Syllabus)
MM223 Topology – II (Previous Syllabus)
MM224 Partial Differential Equation and Calculus of Variation(New Syllabus)

SEMESTER– III

MM231 Complex Analysis (Previous Syllabus Complex Analysis- I)
MM232 Functional Analysis – I (Revised Syllabus)
MM233 Elective (One among the following)

Automata Theory (Previous Syllabus)
Probability Theory(Previous Syllabus)
Operations Research (New Syllabus)
Algebraic Topology (Previous Syllabus)
Numerical Analysis with Python(New Syllabus)
Algebraic Geometry(New Syllabus)

MM234 Elective (One among the following)

Approximation Theory (Previous Syllabus)
Curves, Surfaces and Manifolds(New Syllabus)
Geometry of Numbers (Previous Syllabus)
Differential Geometry (Revised Syllabus)
Graph Theory (Previous Syllabus)
Fractal Geometry (New Syllabus)

SEMESTER – IV

MM241 Number Theory (New Syllabus)
MM242 Functional Analysis – II (Revised Syllabus)
MM243 Elective (One among the following)

Mathematical Statistics (Previous Syllabus)

Difference Equations (Previous Syllabus)
Theory of Wavelets (Previous Syllabus)
Coding Theory (New Syllabus)
Advanced Algebra (New Syllabus)
Mechanics (Previous Syllabus)
Cryptography (New Syllabus)

MM244 Elective (One among the following)

Advanced Graph Theory (Previous Syllabus)
Commutative Algebra (Previous Syllabus)
Advanced Complex Analysis(New Syllabus-previous syllabus of Complex Analysis II)
Representation Theory of Finite Groups (Previous Syllabus)
Category Theory (Previous Syllabus)
Spectral Graph Theory(New Syllabus)

MM 211 LINEAR ALGEBRA

Text

Sheldon Axler, Linear Algebra Done Right 2nd Edition, Springer.

Unit I

Vector spaces: Definition, Examples and properties, Subspaces, Sum and Direct sum of subspaces, Span and linear independence of vectors, Definition of finite dimensional vector spaces, Bases: Definition and existence, Dimension Theorems.
[Chapters 1,2 of Text]

Unit II

Linear maps, their null spaces and ranges, Operations on linear maps in the set of all linear maps from one space to another , Rank-Nullity Theorem , Matrix of linear map, its invetibilty.
[Chapter 3 of Text]

Unit III

Invariant subspaces, Definition of eigen values and vectors, Polynomials of operators, Upper triangular matrices of linear operators, Equivalent condition for a set of vectors to give an upper triangular operator, Diagonal matrices, Invariant subspaces on real vector spaces. [Chapter 5 of Text]

Unit IV

Concept of generalized eigen vectors, Nilpotent operators, Characteristic polynomial of an operator, Cayley-Hamilton theorem, Condition for an operator to have a basis consisting of generalized eigen vectors, Minimal polynomial. Jordan form of an operator (General case of Cayley-Hamilton Theorem may be briefly sketched from the reference text) [Chapter 8 of Text]

Unit V

Change of basis, trace of an operator, Showing that trace of an operator is equal to the trace if its matrix, determinant of an operator, invertibilty of an operator and its determinant, relation between characteristic polynomial and determinant, determinant of matrices of an operator w.r.t. two base are the same. Determinant of a matrix (except the section volumes) [Chapter 10 of Text]

References

1.Kenneth Hoffman and Ray Kunze, Linear Algebra, Prentice Hall, 1981.
2.I.N Herstein, Linear Algebra, Wiley Eastern.
3.S. Kumaresan, Linear Algebra, Prentice Hal, 2000.

MM 212 REAL ANALYSIS-I

Text: Tom M. Apostol, Mathematical Analysis, Second Edition, Narosa 1974

UNIT I

Functions of Bounded Variation and Rectifiable Curves: Properties of monotonic functions, Functions of bounded variation, Total variation, Additive property of total variation, Total variation on [a,x] as a function of x, Function of bounded variation expressed as the difference of increasing functions, Continuous functions of bounded variation, Curves and paths, Rectifiable paths and arc-length, Additive and continuity of arc length, Equivalence of paths, Change of parameter. [Chapter 6 of Text]

UNIT II

The Riemann-Stieltjles Integral: The definition of Riemann-Steiltjles integral, Linear properties, Integration by parts, Change of variable in a Riemann –Stieltjes integral, Reduction to a Riemann integral, Step functions as integrators, Reduction of a Riemann-Stieltjes integral to a finite sum, Euler's summation formula, Monotonically increasing integrators, Upper and lower integrals, Additive and linearity properties of upper and lower integrals, Riemann's condition, Comparison Theorems, Integrators of bounded variation, Sufficient conditions for the existence of Riemann-Stieltjes integrals, Differentiation under the integral sign. [Chapter 7, Sections 7.1-7.16,7.24 of Text]

UNIT III

Sequences of Functions: Point-wise convergence of sequences of functions, Examples of sequences of real-valued functions, Definition of uniform convergence, Uniform convergence and continuity. The Cauchy condition for uniform convergence, Uniform convergence of infinite series of functions, Uniform convergence and Riemann-Stieltjes integration, Non-uniformly convergent series that can be integrated term by term, Uniform convergence and differentiation, Sufficient conditions for uniform convergence of a series.
[Chapter 9, Sections 9.1-9.9 except 9.7 of text . Do suficient problems to study the uniform convergence of sequences and series]

UNIT IV

Multivariable differential Calculus: The directional derivative, Directional derivative and continuity, the total derivatie, total derivative interms of partial derivatives, the matrix of a linear function, the Jacobian matrix, the chain rule, matrix form of the chain rule, the mean value theorem for differentiable functions, sufficient condition for differentiability, sufficient condition for equality of mixed partial derivatives, Taylor's formula for functions from R^n to R Chapter 12(all sections except 12.6) of the text book.

UNIT V

Implicit Functions and Extremum problems: Functions with nonzero Jacobian determinant, the inverse function theorem, the implicit function theorem, Extrema of real valued functions of several variables, extremm problems with side conditions. Chapter 13(all sections) of the text.

References:

1. **J.A Dieudonne, Foundations of Modern Analysis, Academic Press**
2. **W.Rudin, Principles of Mathematical Analysis, Third Edition**
3. **Tom M Apostol, Calculus, Volume 1, Wiley Edition.**
4. **Tom M Apostol, Calculus, Volume 2, Wiley Edition.**

MM213 ORDINARY DIFFERENTIAL EQUATIONS

TEXT: **Differential Equations with Applications and Historical Notes, Simmons G.F, Third Edition, CRC Press, 2017**.

Unit I

The Method of Successive Approximations, Picard's Theorem, Systems. The Second Order Linear Equation, A Review of Power Series, Series Solutions of First Order Equations, Second Order Linear Equations, Ordinary Points, Regular Singular Points, Regular Singular Points (Continued), Two Convergence Proofs. [Chapter 13(sections 69, 70 and 71) and Chapter 5(sections 26, 27, 28, 29, 30 and Appendix A)]

Unit II

Gauss's Hypergeometric Equation, The Point at Infinity, Legendre Polynomials, Properties of Legendre Polynomials [Chapter 5 (sections 31 and 32), Chapter 8(sections 44 and 45)]

Unit III

Bessel Functions. The Gamma Function, Properties of Bessel Functions, Additional Properties of Bessel Functions[Chapter 8(sections 46 and 47; Appendix C)]

Unit IV

Systems of First Order Equations: General Remarks on Systems, Linear Systems, Homogeneous Linear Systems with Constant Coefficients , Nonlinear Systems, Volterra's Prey-Predator Equations[Chapter 10 (sections 54, 55, 56 and 57)]

Unit V

Nonlinear Equations: Autonomous Systems, The Phase Plane and its Phenomena, Types of Critical Points, Stability , Critical Points and Stability for Linear Systems, Stability by Liapunov's Direct Method , Simple Critical Points of Nonlinear Systems[Chapter 11(sections 58, 59, 60, 61 and 62)]

References

[1] G. Birkhoff and G.C. Rota: Ordinary Differential Equations(3rd Edn.); Edn. Wiley & Sons;1978

[2] W.E. Boyce and R.C. Diprima: Elementary Differential Equations and boundary value problems(2nd Edn.); John Wiley & Sons, NY; 1969

[3] A. Chakrabarti: Elements of Ordinary Differential Equations and special functions; Wiley Eastern Ltd., New Delhi; 1990

[4] E.A. Coddington: An Introduction to Ordinary Differential Equtions; Printice Hall of India,New Delhi; 1974

[5] R.Courant and D. Hilbert: Methods of Mathematical Physics- vol I; Wiley Eastern Reprint;1975

[6] P. Hartman: Ordinary Differential Equations; John Wiley & Sons; 1964

[7] L.S. Pontriyagin : A course in Ordinary Differential Equations Hindustan Pub. Corporation,Delhi; 1967

MM 214: TOPOLOGY I

Text book:
Principles of Topology by Fred H. Croom, Baba Barkha Nath Printers (India), Third Reprint 2009.

In this course we discuss the basics of topology , based on chapters 3 and 6. Students should be motivated as discussed in the first two chapters of the Text book.

Unit I

Metric Spaces:-Definition, Examples, Open sets, Closed sets, Interior, closure and boundary
Sections 3.1, 3.2, 3.3

Unit II

Continuous functions, Equivalence of metric spaces, Complete metric spaces-Cantor's Intersection Theorem.
Sections 3.4, 3.5, 3.7, Exercise 3.7(3).

Unit III

Topological spaces:-Definition, Examples, Interior, Closure, Boundary, Base, Sub base, Continuity, Topological Equivalence, Subspaces.
Sections 4.1, 4.2, 4.3, 4.4, 4.5

Unit IV

Connectedness and disconnected spaces, Theorems on connectedness, Connected subsets of real line, Applications of connectedness, Path connected spaces.
Sections 5.1, 5.2, 5.3, 5.4, 5.5

Unit V

Compact spaces, compactness and continuity, Properties related to compactness, One point compactification.
Sections 6.1, 6.2, 6.3,6.4

References

1. Gerald Buskes, Arnoud van Rooiji, Topological Spaces from Distance to Neighbourhood

2. James R. Munkres, Topology, PHI Learning Private Limited, Second Edition, 2009.

3. Stephen Willard, General Topology, Addison-Wesley, Reading, 1970.

4. G.F. Simmons, Topology and Modern Analysis, Mc Graw-Hill, New York, 13th reprint, 2010.

5. J Arthur Seebach, Lynn Arthur Steen, Counter Examples in Topology, Dover Publications, 1995.

6. Sheldon W. Davis, Topology, Tata Mc Graw-Hill Edition, 2006.

MM221 ABSTRACT ALGEBRA

Text: J A Gallian, Contemporary Abstract Algebra, 8thEdition, Cengage Learning.

Unit I

External direct product of groups – Definition and examples, Properties, Representing groups of units modulo n as an external direct product. Normal subgroups and Factor groups, Application of factor groups, Internal direct products, Fundamental theorem of abelian groups, Isomorphism classes, Proof of the fundamental theorem. (Chapter 8, 9 and 11).

Unit II

Sylow theorems, Conjugacy classes, Class Equation, Sylow theorems and Applications. Simple groups, Examples, Non simplicity tests. (Chapter 24 and 25). Theorems 25.1, 25.2 and 25.3 and corollary 1(Index Theorem), corollary 2 (embedding Theorem) may be discussed without proof.

Unit III

Extension fields, Fundamental Theorem of Field Theory, Splitting fields, Zeros of irreducible polynomial, Perfect field. Algebraic extensions, Characterization of extensions, Finite extensions, Properties of algebraic extensions. (chapter 20 and 21).

Unit IV

Finite fields, Classification of finite fields, Structure of finite fields, Subfields of a finite field. Historical discussion of Geometric Constructions, Constructible Numbers, Angle – Trisectors and Circle – Squarers. (Chapter 22 and 23).

Unit V

Fundamental theorem of Galois Theory(without proof), Solvability of polynomials by radicals, Insolvability of Quintic. Cyclotomic polynomials, The Constructible regular n - gons (Chapters 32 and 33).

References:

1. J. B. Fraleigh, *A first course in Abstract Algebra*, Seventh Edition, Pearson Education Inc.

2. I. N. Herstein, *Topics in Algebra*, Second Edition, Wiley

3. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, *Basic Abstract Algebra*, 2 nd Edition, U.K., Cambridge University Press, 2004.

4. David. S. Dummit, and Richard M.Foote, *Abstract Algebra*, 3rd Edition, New Delhi, Wiley, 2011.

MM 222 REAL ANALYSIS II

Text: G de Barra, Measure Theory and Integration, New Age International Publishers, Second Edition 2013.

Unit I

Lebesgue Outer Measure, Measurable sets, Regularity, Measurable functions, Borel and Lebesgue Measurability (Chapter 2, sections 2.1 to 2.5 of Text).

Unit II

Integration of non – negative functions, The General Integral, Integration of Series, Reimann and Lebesgue Integrals, The four Derivatives, Lebesgue Differentiation Theorem(theorem7 and 8 -statement only), Differentiation and Integration.

(Chapter 3, sections 3.1 to 3.4, Chapter 4, sections 4.1, 4.4 (statements only), 4.5 (up to theorem 15) of Text).

Unit III

Abstract Measure Spaces, Measure and outer Measure, Extension of a Measure, Uniqueness of extension, Completion of Measure, Measure spaces, Integration with respect to a Measure. (Chapter 5, sections 5.1 to 5.6 of Text).

Unit IV

The L^p spaces, Convesx functions, Jensen' Inequality, The Inequalities of Holder and Minkowski, Completeness of $L^p(\mu)$.

(Chapter 6, sections 6.1 to 6.5 of Text).

Unit V

Convergence in Measure, Signed Measures and Hahn Decomposition, The Jordan Decomposition, The Radon – Nikodym Theorem.

(Chapter 7 section 7.1, Chapter 8, sections 8.1 to 8.3).

References:

1. H L Royden, P M Fitzpatrik, Real Analysis, Fourth Edition, Pearson, 2017.

2. W Rudin, Real and Complex Analysis, Third Edition, Tata Mc – Graw Hill Edition 2017.

3. P R Halmos, Measure Theory, Springer

4. Measure and Integration: A First Course, M Thamban Nair, First Edition, Chapman and Hall/CRC, 2019.

MM 223: TOPOLOGY II

Text book I: Principles of Topology by Fred H. Croom, Baba Barkha Nath Printers (India), Third Reprint 2009.

Text book II: Topology by Sheldon W. Davis, Tata Mc Graw-Hill Edition, 2006.

Unit I

Product and Quotient spaces:- Finite and arbitrary products, Comparison of topologies, Quotient spaces.

Sections 7.1, 7.2, 7.3, 7.4 of Text I, (Alexander sub basis theorem and Theorem 7.11 excluded).

Unit II

Separation axioms:-T_0;T_1 and T_2-spaces, Regular spaces, Normal spaces, Separation by continuous functions

Sections 8.1, 8.2, 8.3, 8.4 of Text I

Unit III

Convergence, Tychnoff 's Theorem

Chapter 16, Theorem 18.21 and Theorem 18.22 of Text II

Unit IV

Algebraic topology:- The fundamental group, The fundamental group of S^1.

Sections 9.1, 9.2, 9.3 of Text I

Unit V

Examples of fundamental groups, The Brouwer Fixed Point Theorem.

Sections 9.4 and 9.5 of Text I

References

1.Gerald Buskes, Arnoud van Rooiji, Topological Spaces from Distance to Neighbourhood

2.2.James R. Munkres, Topology, PHI Learning Private Limited, Second Edition, 2009.

3. Stephen Willard, General Topology, Addison-Wesley, Reading, 1970.

4.G.F. Simmons, Topology and Modern Analysis, Mc Graw-Hill, New York, 13th reprint, 2010.

5. J Arthur Seebach, Lynn Arthur Steen, Counter Examples in Topology, Dover Publications, 1995.

6.Sheldon W. Davis, Topology, Tata Mc Graw-Hill Edition, 2006.

MM 224 PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Text 1: An introduction to partial differential equations, Yehuda Pinchover and Jacob, Cambridge University Press, 2005

Text 2: Methods of Applied Mathematics (second Edition), Hildebrand F.B, Prentice-Hall of India, New Delhi, 1965.

Text 3: Differential equations with applications and historical notes(third edition),George F. Simmons, McGraw-Hill International Edn, 2017

Unit I

Introduction : Preliminaries, classification, differential operators and the superposition principle.
First-order equations: Introduction, quasilinear equations, the method of characteristics, examples of the characteristics method, the existence and uniqueness theorem, the Lagrange method, General nonlinear euations[Chapter 1(sections 1.1 to 1.3), chapter 2 (sections 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.9, 2.10 from Text 1]

Unit II

Second-order linear equations in two indenpendent variables: Introduction, classification, canonical form of hyperbolic equations, canonical form of parabolic equations, canonical form of elliptic equations, the one-dimensional wave equation: Introduction, canonical form and general solution, the Cauchy problem and d'Alemberts formula, domain of dependence and region of influence, the Cauchy problem for the nonhomogeneous wave equation [Chapter 3, 4 from Text 1]

Unit III

The method of separation of variables: Introduction, heat equation: homogeneous boundary condition, Separation of variables for the wave equation, Separation of variables for nonhomogeneous equations, The energy method and uniqueness, Further applications of the heat equation Elliptic equations: Introduction, Basic properties of elliptic problems, The maximum principle, Applications of the maximum principle, Greens identities, The maximum principle for the heat equation, Separation of variables for elliptic problems, Poissons formula.[Chapter 5 and 7 from Text 1]

Unit IV

Integral Equations: Introduction, Relations between differential and integral equations,

The Green's functions, Fredholom equations with separable kernels, Illustrative examples,

Hilbert- Schmidt Theory, Iterative methods for solving Equations of the second kind. The

Newmann Series, Fredholm Theory [chapter 3(Sections 3.1, 3.2, 3.3, 3.6, 3.7, 3.8, 3.9, 3.10, 3.11) from the Text 2]

Unit V

Calculus of variations: Introduction, some typical problems of the subject, Euler differential equation for an extremal, isoperimetric problems [Chapter 12 (sections 66, 67, and 68) from Text 3]

References:

[1] Amaranath T.:Partial Differential Equations, Narosa, New Delhi, 1997.

[2] A. Chakrabarti: Elements of ordinary Differential Equations and special functions;

Wiley Eastern Ltd, New Delhi; 1990

[3] E.A. Coddington: An Introduction to Ordinary Differential Equations Printice Hall

of India ,New Delhi; 1974

[4] R. Courant and D. Hilbert: Methods of Mathematical Physics-Vol I; Wiley Eastern

Reprint; 1975

[5] P. Hartman: Ordinary Differential Equations; John Wiley & Sons; 1964

MM 231 COMPLEX ANALYSIS

Text: John. B. Conway, Functions of Complex Variables, Springer-Verlag, New York, 1973. (Indian Edition: Narosa)

UNIT I

Elementary properties and examples of analytic functions, Power series, Analytic function, Riemann- Stieltjes integrals.

(Chapter 3- Sections 1, 2 and Chapter 4- Section 1 of Text)

UNIT II

Power series representation of an analytic function, Zeros of an analytic function, The index of a closed curve.

(Chapter 4 – Sections 2, 3 and 4 of Text)

UNIT III

Cauchy's Theorem and integral formula, Homotopic version of Cauchy's Theorem, Simple connectivity, Counting zeros: The open Mapping Theorem, Goursat's Theorem.

(Chapter 4 - Sections 5, 6, 7 and 8 of Text)

UNIT IV

Singularities: Classification, Residues, The argument principle.

(Chapter 5 Sections 1, 2, and 3 of Text)

UNIT V

The extended plane and its spherical representation, Analytic function as mapping, Mobius transformations, The maximum principle, Schwarz's Lemma.

(Chapter 1- Section 6, Chapter 3- Section 3, Chapter 6- Section 1 and 2 of Text)

References:

1. L.V. Ahlfors, Complex Analysis, Mc-Graw Hill (1966)

2.S. Lang, Complex Analysis, Mc-Graw Hill (1998).

3.S. Ponnusamy & H. Silverman, Complex Variables with Applications, Birkhauser

4.H. A. Priestley, Introduction to Complex Analysis, Oxford University Press Tristan Needham, Visual Complex Analysis, Oxford University Press(1999)

5.V. Karunakaran, Complex Analysis, Narosa Publishing House, 2002

MM 232: FUNCTIONAL ANALYSIS- I

Text: B. V. Limaye, FUNCTIONAL ANALYSIS (3rd Edition)

A quick review of chapter I of the Text is to be done as a prerequisite to the Functional Analysis course.

UNIT I

Normed spaces and continuity of linear maps. (Section 5 and 6 of the Text, Except 6.5 (d) and Theorem 6.8).

UNIT II

Hahn-Banach theorems and Banach spaces. (Section 7 and 8 of the Text,Theorem7.12 statement only).

UNIT III

Uniform boundedness principle, closed graph and open mapping theorems (Section 9.1, 9.2, 9.3 and 10 of the Text).

UNIT IV

Bounded inverse theorem, spectrum of a bounded operator (Section 11.1, 11.3, 12 (Except 12.4) and 13.1 of the Text,).

UNIT V

Transpose-Definition as in Section 13 and Theorem13.5, Weak convergence, reflexivity and compact linear maps (Sections 15.1, 15.2 (a), 16.1, 16.2, 17.1, 17.2 and 17.3, 17.4 (a) of the Text).

References

1.Bryan Rynne, M.A. Youngson, Linear Functional Analysis, Publisher: Springer.

2.Rajendra Bhatia, Notes on Functional Analysis, Publisher: Hindustan Book Agency.

3.M. Tamban Nair, Functional Analysis: A first course, Publisher: Prentice Hall of India Pvt. Ltd.

4.Walter Rudin, Functional Analysis, 2nd Edition, Publisher: Tata Mc Graw-Hill.

5.B. V. Limaye, Linear Functional Analysis for Scientists and Engineers, Springer Singapore, 2016.

MM 233 AUTOMATA THEORY (Elective)

Text: J.E. Hopcroft and J.D. Ullman, *Introduction to Automata Theory, Languages and Computation*, Narosa, 1999

UNIT I

Strings, AlphabetsandLanguages (Section 1.1 of the Text)
Finite Automata (Chapters 2, Sections 2.1 to 2.4)

UNIT II

Regular expressions and Properties of Regular sets.(Sections 2.5 to 2.8 and 3.1 to 3.4)

UNIT III

Context Free grammars (Section 4.1 to 4.5)

UNIT IV

Pushdown Automata & properties of Context free languages
Theorem 5.3, 5.4 (without proof), (Section is 5.1 to 5.3 and 6.1 to 6.3)

UNIT V

Turning Machine and Chomski hierarchy, (Sections 7.1 to 7.3 and 9.2 to 9.4)

References

1. J.E. Hopcroft, R. Motwani and J.D. Ullman, *Introduction to Automata Theory, Languages and Computation, 3rd edition, Pearson, 2008.*
2. P.Linz, An i*ntroduction to Formal Languages and Automata, 6h edition, Jones & Bartlettudent Edition, 2012.*
3. K. L. P. Mishra, N. Chandrasekharan, Theory of computer science: Automata languages and computation, PHI, 2006.
4. G.ERevesz, *Introduction to Formal Languages, Dover, 2012.*5. M. Sisper, Introduction to the theory of computation, 2nd edition, Thomson, 2006.

MM 233 PROBABILITY THEORY(Elective)

Texts:

(1) **Laha R.G and Rohatgi V.K ,“ *Probability Theory*”, JohnWiley, New York (1979)**

(2) **Johnson N.L and KotzS “*Distributions in Statistics: Discrete Distributions*”, John Wiley, New York (1969)**

(3) **Johnson N.L, and Kotz S “ *Distributions in Statistics: Continuous Univariate Distributions*”, Vol 1 and 2 ,John Wiley, New York (Paperback, 1970)**

UNIT I

Probability,liminf,limsup,and limitofsequence of events, Monotone and continuity property of probability measure, Addition Theorem, Independence of finite number of events, Sequence of events, Borel Cantalls Lemma, Borel Zero one law

UNIT II

Random variable, Its probability distribution function, Properties of distribution function, Discrete and continues type random variables, Discrete, Continuous and other types of distributions, Expectation and moments of random variables, Inequalities of Liaponov (for moments) , Random vectors, Independence of random variables and sequence of random variables, Markov and Chebychev’s inequalities.

UNIT III

Standard distributions and their propertiesBernoulli, Binomial, Geometric, Negative Binomial, Hyper geometric, Beta, Cauchy, Chi square, Double Exponential, Exponential, Fisher’s F, Gamma, Log Normal, Normal,, Parents, Students’s t, Uniform and Heibull.

UNIT IV

Characteristic functions and their elementary properties, Uniform continuity and non negative definiteness of characteristic functions, Characteristic functions and moments, Statement (without proof) and application of each of the three theorems Inversion Theorem, Continuity Theorem and Bochner Khintchine Theorem of characteristic functions, Statement and proof of Fourier Inversion Theorem.

UNIT V

Stochastic convergence of sequence of random variables, Convergence in distributions, Convergence in probability, Almost sure convergence and convergence in the rth mean, Their inter-relation ship - Examples and counter examples, Slutsky’s Theorem.

References:

(1). Ash R.B- “*Basic Probability Theory*”, John wiley, New York (1970)

(2). Bhat B.R-“*Modern Probability Theory: An Introduction Text Book*”, Wiley Eastern (Second Edition) 1985

(3). Gnedenko B.V-"*The Theory of Probability*",
Mir Publishers Moscow (1969)
(4). Luckacs.E-"*Characteristic Functions*", Hafner, New York (Second Edition, 1970)
(5). Luckacs. E- "*Stochastic Convergence*", Academic Press (Second Edition, 1975)
(6). Johnson N.L, Kots,S and Balakrishnan.N-"*Continuos Univariate Distribution*",
***Vol.1 and 2* , John Wiley, New York (Second Edition, 194 vol.1 1995 Vol.2)**
(7). Johnson N.L, Kots.S and Kemp A.W-"*Univariate Discrete Distributions*",
JohnWiley, New York" Second Edition 1992)

MM 233 OPERATIONS RESEARCH (Elective)

Text 1 : J K Sharma. Operations Research - Theory and Applications, Sixth Edition, 2016

Text 2 :

K V Mital, C Mohan. Optimization Methods in OperationResearch and System analysis, Third Edition, New Age International Publishers, New Delhi.

Unit I

Linear Programming - Definitions, Graphical solution methods for LPproblems.
Simplex Method - Standard form of an LP problem, simplex algorithm(Maximizationcase), simplex algorithm(Minimization case)- Two Phase method, Big-MMethod
Chapter 3 of text 1 - sections 3.1 to 3.3, Chapter 4 of text 1 - sections 4.1 to4.4

Unit II

Transportation Problem : Mathematical model of TP, The transportaionalgorithm, Methods for finding initial solution, Test for optimality Assignment Problems : Mathematical model of AP, Solution methods of AP Chapter 9 of text 1 - Section 9.1,9.2, 9.3, 9.4, 9.5; Chapter 10 of text 1 - Section 10.1, 10.2, 10.3.

Unit III

Project Management : Basic differences between PERT and CPM, Phases of Project Management, PERT/CPM network components and precedence relationships, Critical path analysis Chapter 13 of text 1 - Section 13.1 to 13.5

UNIT IV

Kuhn Tucker Theory and Nonlinear Programming: Lagrangian function, saddle point, Kuhn Tucker conditions, Primal and dual problems, Quadratic Programming.
[Chapter8 of text 2, sections 1 to 6]

UNIT V

Dynamic Programming: Minimum path, Dynamic Programming problems, Computational economy in DP, serial multistage model, Examples of failure, Decomposition, Backward recursion.
[Chapter 10 of text 2, sections 1 to 10]

Reference:

Hamdy A. Taha, *Operations Research*, Fifth edition, PHI

MM 233 ALGEBRAIC TOPOLOGY (Elective)

Text Book : **Basic Concepts of Algebraic Topology, Fred H. Croom** Springer – Verlang

Unit I

Introduction, Examples, Geometric Complexes and Polyhedra, Orientation of Geometric complexes. (Sections 1.1, 1.2, 1.3, 1.4 of Chapter 1)

Unit II

Simplicial Homology Groups - Chains, cycles, Boundaries, Homology groups, examples of Homology Groups, The structure of Homology Groups, The Euler Poincare Theorem, Psudo manifolds and the Homology Groups of $S^{n.}$ (Sections 2.1, 2.2, 2.3, 2.4, 2.5 of chapter 2).

Unit III

Simplicial Approximation- Introduction, Simplicial Approximation, Induced Homomorphisms on the Homology groups, the Browerfixed point theorem and related results .
(Sections 3.1, 3.2, 3.3, 3.4 of Chapter 3)

Unit IV

The Fundamental group - Introduction, Homotopic paths and the fundamental group, the covering homotopy property for $S^{1,}$ Examples of Fundamental group, the relation between $H_1(K)$ and $\pi_1([K])$.
(Sections 4.1,4.2,4.3, 4.4 and 4.5 of chapter 4)

Unit V

Covering Spaces – Definition and examples, basic properties of Covering spaces, Classification of covering spaces, Universal covering spaces, and applications
(Section 5.1, 5.2, 5.3, 5.4 and 5.5 of Chapter 5)

References :

(1) I.M Singer, J.A Thorpe, Lecture Notes on Elementary Topology and Geometry, Springer International Edition, Springer (India) Private Limited, New Delhi, 2004

(2) Satya Deo, Algebraic Topology A Primer, Hindustan Book Agency, New Delhi, 2003.

(3) Allen Hatcher, Algebraic Topology , Published 2001 by Cambridge University Press

MM 233 NUMERICAL ANALYSIS WITH PYTHON(Elective)*

Texts:

1.Vernon L. Ceder, *The Quick Python Book*, Second Edition, Manning.

2.Richard L. Burden and J. Douglas Faires, *Numerical Analysis*, Ninth Edition,

Brooks/Cole, Cengage Learning.

3.Amit Saha, *Doing Math with Python,* No Starch Press, 2015.

Unit I

In this unit we discuss the basics of python based on chapters 4,5,6,8, 9, 10 and 18 of Text 1. All topics of chapters 4-9 must be dicussed using exmples from mathematics. In chapter 10, only sections 10.1-10.4 need to be discussed. Chapter 18 also shoud be discussed to get an overview of the packages in python. The students should be encouraged to write programs related with mathematical problems. (Some of the problmes are listed in the syllabus)

Unit II

Visualizing Data with Graphs - learn a powerful way to present numerical data: by drawing graphs with Python. The unit is based on Chapter 2 of Text 3. The sections Creating Graphs with Matplotlib and Plotting with Formulas must be done in full. In the section Programming Challenges, the problems Exploring a Quadratic Function Visually, Visualizing Your Expenses and Exploring the Relationship Between the Fibonacci Sequence and the Golden Ratio must also be discussed.

Unit III

The unit is based on chapters 4 and 7 of Text 3. Here we discuss Algebra and Symbolic Math with SymPy and Solving Calculus Problems. In Chapter 4 the sections Defining Symbols and Symbolic Operations, Working with Expressions, Solving Equations and Plotting Using SymPy should be done in full. In the section Programming Challenges, the problems Factor inder, Graphical Equation Solver, Summing a Series and Solving Single-Variable Inequalities also should be discussed. In chapter 7, some problems discussed namely, Finding the Limit of Functions, Finding the Derivative of Functions, Higher-Order Derivatives and Finding the Maxima and Minima and Finding the Integrals of Functions are to be done. In the section Programming Challenges, the problems Verify the Continuity of a Function at a Point, Area Between Two Curves and Finding the Length of a Curve also should be discussed.

Unit IV

In this unit we discuss some numerical methods for solving system of linear equations, for finding roots of equations and polynomial interpolation from Text 2.We first discuss bisection method, methods of Newton, secant method and method of false position for solving equations of the form *f(x)=0.* The topics can be found in sections 2.1 (only upto example 2) and 2.3. Interpolation and the

Lagrange Polynomial as per section 3.1 (only upto example 2) is to be discussed. Next we discuss Gauss Elimination with backward substitution method and LU decomposition method as per sections 6.1 (only upto Algorithm 6.1) and 6.5 (Theorem 6.19 statement only and exclude subsection Permutation Matrices. Also avoid the discussion of matrix factorization using Maple). Students should be encouraged to do problems and write python program for each method (see ref 1).

Unit V

Here we discuss some numerical methdos for integration, differentiation and solving initial value problems of ordinary differential equations from Text 2. The methods for approximating first drivative of a function as per section 4.1 are to be discussed. They include forward-difference formula, *(n+1)*-point formula, in particular three-point formulas. Rest of the toipics in this section need not be discussed. Next we discuss the methods for numerical integration. Trapezoidal rule and simposns rules are to be discussed from section 4.3. Then we discuss *(n+1)-point closed Newton-Cotes formula* and derive trapezoidal, Simpson's rule and Simpson's 3/8 rules from it. Remaining topics in the section need not be done. We also discuss *Composite Simpson’s rule* and *Composite Trapezoidal rule* from section 4.4 (theorems 4.4 and 4.5 only). Our discussion about numerical methods for solving initial value problems of ordinary differential equations include Euler's method, Runge-Kutta methods of second and fourth order. The topics can be found in sections 5.2 (excluding subsection Error Bounds for Euler’s Method) and section 5.4 (only Midpoint Method for Runge-Kutta Methods of order two and Runge-Kutta method of order four need to be discussed without any proof). Students should be encouraged to do problems and write python program for each method (see ref 1).

Some problems for Unit I are listed below

- Factorial of a number
- Checking primality of a number
- Listing all primes below a given number
- Prime factorization of a number
- Finding all factors of a number
- gcd of two numbers using the Euclidean Algorithm
- Finding the multiples in Bezout’s Identity
- checking the convergence and divergence of sequences and series.

(For more problems visit https://www.nostarch.com/doingmathwithpython/, https://doingmathwithpython.github.io/author/amit-saha.html and https://projecteuler.net/.)

- The course is aimed to give an introduction to mathematical computing with Python as tool for computation.
- The students should be encouraged to write programs to solve the problems given in the sections as well as in the exercises.
- The end semester evaluation should contain a theory and a practical examinations.
- The duration of the theory examination will be 3 hours, with a maximum of 50 marks.
- In the question papers for the theory examination, importance should be given to the definition, concepts and methods discussed in each units, and not for writing long programs.
- Practical examination shall also be of 3 hours duration for a maximum of 25 marks.
- Weightage of marks for theory and a practical examinations is listed below

<table>
<tr><th>Unit</th><th>Theory</th><th>Practical</th></tr>
<tr><td>I</td><td rowspan="3">10
(one question out of two)</td><td>10 (two questions out of four)</td></tr>
<tr><td>II</td><td>5 (one questoin out of two)</td></tr>
<tr><td>III</td><td>5 (one questoin out of two)</td></tr>
<tr><td>IV</td><td>20 (two questoins out of four)</td><td rowspan="2">5
one questoin out of two (one from each unit)</td></tr>
<tr><td>V</td><td>20 (two questoins out of four)</td></tr>
</table>

- Continuous evaluation follows the pattern - 5 marks for attendence, 10 marks for the internal examination and 10 marks for the practical record. The record should contain at least 20 programs.
- The practice of writing the record should be maintained by each student throughout the course and it should be dually certified by the teacher in charge/internal examiner and evaluated by the external examiner of practical examination.

References:

1. Jaan Kiusalaas, *Numerical Methods in Engineering with Python3*, Camdbridge University Press, 2013.

2. *NumPy Reference Release 1.17.0,* Written by the NumPy community. (available at https://docs.scipy.org/doc/)

3. https://docs.python.org/3/tutorial/

4. Ward Cheney and David Kincaid, *Numerical Mathematics and Computing*, Sixth Edition, Thomson Brooks/Cole.

5. R. W. Hamming, *Numerical Methods for Scientists and Engineers*, Second Edition, Dover Publications Inc.

MM 233 ALGEBRAIC GEOMETRY (Elective)

Text Book

1. Elementary Algebraic Geometry, K. Hulek (translated by H. Verrill), Student Mathematical Library, vol 20, American Mathematical Society, 2003.

Unit I

Introduction, affine varieties, Hilbert's Nullstellensatz, polynomial functions and maps.

Unit II

Rational functions and maps, projective space, projective varieties, rational functions and morphisms.

Unit III

Smooth and singular points, algebraic characterizations of the dimension of a variety, plane curves,

Unit IV

Intersection multiplicity, classification of smooth cubics, the group structure of an elliptic curve.

Unit V

The existence of lines on a cubic, configuration of the 27 lines, rationality of cubics.

References

1. Introduction to Algebraic Geometry, Brendan Hassett, Cambridge University Press, 2007.
2. Algebraic Geometry, R. Hartshorne, Springer-Verlag, 1977.
3. Algebraic Geometry: A First Course, J. Harris, Springer-Verlag, 1992.
4. Algebraic Curves: An Introduction to Algebraic Geometry, William Fulton, Advanced Book Program, Redwood City, Addison-Wesley, 1989.
5. Principles of Algebraic Geometry, Phillip Griffiths and Joseph Harris, New York: Wiley-Interscience, 1978.

MM 234APPROXIMATION THEORY (Elective)

Text: EW Cheney, "*Introduction to Approximation Theory*", Mc Graw Hill

UNIT 1

Metric spaces- An existence Theorem for best approximation from a compact subset; Convexity- Caratheodory's Theorem- Theorem on linear inequalities; Normed linear spaces - An existence Theorem for best approximation from finite dimensional subspaces - Uniform convexity - Strict convexity (Sections 1,2,5,6 of Chapter 1)

UNIT II

The Tchebycheff solution of inconsistent linear equations -Systems of equations with one unknown- Three algebraic algorithms; Characterization of best approximate solution for m equations in n unknowns- The special case m=n+1; Poly's algorithm.
(Section 1,2,3,4,5 of Chapter 2)

UNIT III

Interpolation- The Lagrange formula-Vandermonde's matrix- The error formula- Hermite interpolation; The Weierstrass Theorem- Bernstein polynomials- Monotone operators- Fejer's Theorem; General linear families- Characterization Theorem- Haar conditions- Alternation Theorem.
(Sections 1,2,3,4, of Chapter 3)

UNIT IV

Rational approximation- Conversion or rational functions to continued fractions; Existence of best rational approximation- Extension of the classical Theorem; Generalized rational approximation- the characterization of best approximation- An alternation Theorem- The special case of ordinary rational functions; Unicity of generalized rational approximation.
(Sections 1,2,3,4 of Chapter 5)

UNIT V

The Stone Approximation Theorem, The Muntz Theorem - Gram's lemma, Approximation in the mean- Jackson's Unicity Theorem- Characterization Theorem, Marksoff's Therem.
(Section 1,2,6 of Chapter 6)

Reference:

P.J Davis. "*Interpolation and Approximation*", Blaisdell Publications.

MM 234 CURVES, SURFACES AND MANIFOLDS(Elective)

Text Books

1. Elementary Differential Geometry, Andrew Pressley (2^{nd} Edn) Springer-Verlag 2010.
2. Introduction to Smooth Manifolds, John M Lee, Springer-Verlag 2006.

Unit - I

What is a curve, Arc-length, Reparametrization, Closed curves, Level curves versus parametrized curves,
Curvature, Plane curves, Space curves.
[Chapter 1 – Sections 1- 5, Chapter 2 – Sections 1 – 3 **from Text – 1**]

Unit - II

What is a surface, Smooth surfaces, Smooth maps, Tangents and derivatives, Normals and orientability,
Applications of the Inverse Function Theorem.
[Chapter 4 – Sections 1 – 5, Chapter 5 – Section 6 **from Text – 1**]

Unit - III

Lengths of curves on surfaces, The second fundamental form, The Gauss and Weingarten maps, Normal and geodesic curvatures, Gaussian and mean curvatures, Principal curvatures of a surface.
[Chapter 6 – Sections 1, Chapter 7 – Sections 1 – 3, Chapter 8 – Sections 1 – 2 **from Text - 1**]

Unit – IV

Topological Manifolds, Smooth structures, Examples of smooth manifolds,
Smooth functions and smooth maps.
Tangent vectors, Pushforwards, Computations in coordinates. The tangent bundle, Vector fields on manifolds.
[Chapter 1 sections 1, 3, 4, Chapter 2 section 1, Chapter 3 sections 1-3, Chapter 4 sections 1-2 **from Text - 2**]

Unit - V

Covectors, tangent covectors on manifolds, the cotangent bundles, the differential of a function, pullbacks, line integrals.
Maps of constant rank, Embedded submanifolds.
[Chapter 6 sections 1-6, Chapter 7 section 1, Chapter 8 section 1 **from Text - 2**]

References

1. Differential Geometry of Curves and surfaces, M P do Carmo, Prentice-Hall, 1976.
2. A course in Differential Geometry, W Klingenberg, Walter de Gruyter, 1995.
3. Elementary Differential Geometry, D B. O'Neil, Academic Press NY 1966.
4. Calculus on Manifolds, Michael Spivak, Westview Press, 1971.
5. An Introduction to Manifolds, Loring W. Tu, Springer-Verlag (2^{nd} edn.) 2011.
6. Analysis on Manifolds – J R Munkres, Addison-Wesley 1991.
7. Lectures on classical differential geometry, Dirk Jan Struik, Dover Publications (2^{nd} edn.) 1988.
8. A Panoramic View of Riemannian Geometry, Marcel Berger, Springer 2002.

MM 234 GEOMETRY OF NUMBERS (Elective)

Text Book: D.D Olds,Anneli Lax and Guiliana P. Davidoff, *The Geometry of Numbers,* The Mathematical Association of America 2000

UNIT 1

Lattice points and straight lines, Counting of lattice points (Chapters 1 and 2)

UNIT 2

Lattice points and area of polygons, Lattice points in circles (Chapter 3 and 4)

UNIT 3

Minkowski fundamental Theorem and Applications (Chapters 5 and 6)

UNIT 4

Linear transformation and integral lattices, Geometric interpretations of Quadratic forms (Chapters 7 and 8)

UNIT 5

Blichfieldts and applications, Tchebychev's and consequences (Chapter 9 and 10)

References

1. **J.W.S Cassells, *Introduction to Geometry of Numbers,* Springer Verlag 1997**
2. **C.I Siegel, *Lectures in Geometry of Numbers,* Springer Verlag 1989.**

MM 234 DIFFERENTIAL GEOMETRY (Elective)

Text: John.A. Thorpe, *Elementary Topics in Differential Geometry,* Springer Verlag

UNIT I

Graphs and level sets, Vector fields, Tangent Spaces . (Chapter 1,2,3 of Text)

UNIT II

Surfaces, Vector fields on surfaces, Orientation, The Gauss map (Chapter 4,5 6 of Text)

UNIT III

Geodesics, Parallel transport (Chapter 7,8 Text)

UNIT IV

The Weingarten map, Curvature of plane curve. (Chapter 9.10 of Text)

UNIT V

Arc length, Line integral, Curvature of surfaces (Chapter 11,12 of Text, except the proofs of Theorem 1, Theorem 2 of Chapter 11 and Theorem 1 of Chapter 12)

References:

[1] I Singer and J.A Thorpe, *Lecture notes on Elementary Topology and Geometry,* Springer-Verlag

[2] M Spivak, *Comprehensive introduction to Differential Geometry (Vols 1 to 5),* Publish or Perish Boston.

MM 234 GRAPH THEORY (Elective)

Text: Gary Chartrand and Ping Zhang , *Introduction to Graph Theory,* Tata Mc Graw Hill,
Edition 2006

An overview of the concepts-Graphs, Connected graphs, Multi graphs, Degree of a vertex, Degree
Sequence, Trees.

UNIT I

Definition of isomorphism, Isomorphism as a relation, Graphs and groups, Cut-vertices, Blocks,
Connectivity.
(Sections 3.1, 3.2, 3.3, 5.1, 5.2 and 5.3)

UNIT II

Eulerian graphs, Hamilton graphs, Hamilton walks and numbers
(Sections 6.1, 6.2 and 6.3)

UNIT III

Strong diagraphs, Tournaments, matching, Factorization.
(Sections 7.1, 7.2, 8.1,8.2)

UNIT IV

The Four colour problem, Vertex colouring, The Ramsey number of graphs, Turan's Theorem.
(Sections 10.1, 10.2, 10.3, 11.1 and 11.2)

UNIT V

The centre of a graph, Distant vertices, Locating numbers, Detour and Directed distance.
(Sections 12.1, 12.2, 12.3, 12.4)

References:

1. **Bondy J.A and Murthy U.S.R, *"Graph Theory with Applications",* the Macmillan Press Limited.**
2. **Hararay F.,*"Graph Theory"*, Addison-Wesley**
3. **Suesh Singh G.,*"Graph Theory"*, PHI Learning Private Limited**
4. **Vasudev.C , *"Graph Theory Applications".***
5. **West D.B,*"Introduction to Graph Theory"*, PHI Learning Private Limited**

MM 234 FRACTAL GEOMETRY (Elective)

Text

Kenneth Falconer, Fractal Geometry Mathematical Foundation and Application, Third edition, Wiley, 2014

Unit 1

Basic set theory , Functions and limits, Measures and mass distributions, Box-counting dimensions, properties of box –counting dimensions

(sections 1.1,1.2,1.3,2.1,2.2 of text)

Unit II

Hausdorff measure, Hausdorff dimension, Calculation of Hausdorff dimension, Basic method for calculating dimensions.

(sections 3.1,3.2,3.3,4.1 of text)

Unit III

Iterated functions systems, Dimensions of self similar sets, Some variations, Continued fraction examples

(sections 9.1,9.2,9.3,10.2 of text)

Unit IV

Dimensions of Graphs, The Weierstrass function and self –affine graphs, Repellers and iterated function system, The logistic map

(sections 11.1,13.1,13.2 of text)

Unit V

Sketch of general theory of Julia sets, The Mandelbort set, Julia sets of quadratic functions

(sections 14.1,14.2,14.3 of text

References

1. Falconer K.J, The Geometry of Fractal sets ,Cambridge University Press, Cambridge, 1986

2. Barnsley M F, (1988), Fractals Every where, Academic press

MM 244 NUMBER THEORY

Text Book: Tom M. Apostol, *Introduction to Analytical Number Theory*, Springer, 1998

UNIT I

Arithmetical function and Dirichlet multiplication

(Section 2.1 to 2.14 of Text)

UNIT II

Finite Abelian Groups and Their Characters

(Theorem 4.1 of Chapter 4 (Abel's identity), Section 6.5 to 6.10 of Chapter 6 *(Theorem 6.6 Statement Only)*)

UNIT III

Dirichlet's Theorem on Primes in Arithmetic Progressions

(Section 3.2 of Chapter 3, Sections 7.1 to 7.8)

UNIT IV

Quadratic residues, Reciprocity law, Jacobi symbol

(Sections 9.1 to 9.8 of Chapter 9)

UNIT V

Primitive roots, Existence and number of primitive roots.

(Sections 10.1 to 10.9 and Sections 10.11 to 10.13 of Chapter 10)

References

[1] Kenneth Ireland, Michael Rosen, *A Classical Introduction to Modern Number Theory*, Second Edition, Springer 1990.

[2] Emil Grosswald, *Topics from the Theory of Numbers*, Birkhauser 1984

[3] G.H Hardy and E.M Wright , *Introduction to the Theory of Numbers*, Oxford Press.

MM 242: FUNCTIONAL ANALYSIS II

Text: **B. V. Limaye, FUNCTIONAL ANALYSIS (3rd Edition)**

UNIT I

Spectrum of a compact operator (Section 18.1, 18.2, 18.3, 18.4, 18.5 and 18.7 (a) only).

UNIT II

Inner product spaces, orthonormal sets (Section 21 and 22 of the Text, omitting 21.3 (d), 22.3 (b), 22.8 (c), 22.8 (d), 22.8 (e)).

UNIT III

Approximation and optimization, projection and Riesz representation theorems. (Section 23 and 24 of the Text, omitting 23.6).

UNIT IV

Bounded operators and adjoints, normal, unitary and self-adjoint operators (Section 25 and 26.1 to 26.5 of the Text omitting 25.4 (b)).

UNIT V

Spectrum and numerical range, compact self-adjoint operators (Section 27.1,27.2, 27.4(statement only), 27.5, 27.7, 28.1, 28.4, 28.5, 28.6 of the Text).

References

1.Bryan Rynne, M.A. Youngson, Linear Functional Analysis, Publisher: Springer.

2.Rajendra Bhatia, Notes on Functional Analysis, Publisher: Hindustan Book Agency.

3.M. Tamban Nair, Functional Analysis: A first course, Publisher: Prentice Hall of India Pvt. Ltd.

4.Walter Rudin, Functional Analysis, 2nd Edition, Publisher: Tata Mc Graw-Hill.

5.B. V. Limaye, Linear Functional Analysis for Scientists and Engineers, Springer Singapore, 2016.

MM 243 MATHEMATICAL STATISTICS (Elective)

Text: **V.K.Rohatgi**, *An Introduction to Probability Theory and Mathematical Statistics*, Wiley Eastern Ltd.

Unit I: The theory of point estimation:

The problem of point estimation, properties of estimates, unbiased estimation, unbiased estimation (continued): A lower bound for the variance of an estimate, the method of moments, maximum likelihood estimates.(Chapter 8 (Sec 8.2 – sec 8.7) of text)

Unit II: Neyman – Pearson theory of testing of hypothesis:

Introduction, some fundamental notions of hypothesis testing, the Neyman – Pearson Lemma, families with monotone likelihood ration, unbiased and invariant tests.(Chapter 9 of text)

Unit III: Some further result on hypothesis testing:

Introduction, the likelihood ratio tests, the Chi-square tests, the t-tests, the F-tests, Bayes and minimax procedure.(Chapter 10 of text)

Unit IV: Confidence estimation:

Introduction, Some fundamental notions of confidence estimation, shortest length confidence intervals, relation between confidence estimation and hypothesis testing, unbiased confidence intervals, Bayes confidence intervals(Chapter 11 of text)

Unit V: Nonparametric statistical inference:

Introduction, nonparametric estimation, some single – sample problems, some two – sample problems, tests of independence.(Chapter 13 (Sec 13.1 – 13.5) of text)

References:

Lehmann. E.L, "*Theory of Point Estimation*", John Wiley, New York, 1983.

1.**Lehmann E.L**, "*Testing of Statistical Hypothesis*", John Wiley, New York (Second Ed.), 1986.

2.**Randles R H and Wolf D A**, "*Introduction to the Theory of Non parametric Statistics*", Wiley, New York, 1979.

3.**Kendall M G and Stuart A**, "*The Advanced Theory of Statistics*", Vol 2, Mac Millan, New York (Fourth Ed.), 1979.

4.**Mood Ali, Gray Bill, Fhardoes D C**, "*Introduction to the Theory of Satistics*", McGraw Hill International, New York (Third Ed.), 1972.

MM 243 Difference Equations (Elective)

Unit – I: Linear Difference Equations of Higher Order
Difference calculus – General theory of linear difference equations – Linear homogenous equations with constant coefficients – Linear non-homogenous equations – Methodof undetermined coefficients. (Chapter 2: Sections: 2.1 to 2.4)

Unit – II: System of Linear Difference Equation
Autonomous (time invariant) systems –The basic theory –The Jordan form: Autonomous (time-invariant) systems - Linear Periodic Systems. (Chapter 3: Sections: 3.1 to 3.4)

Unit – III: The Z-Transform Method
Definitions and examples – Properties of Z-Transform – The inverse Z-Transform and solutions of difference equations - Power series method - Partial fraction method – Inversion integral method. (Chapter 6: Sections: 6.1,6.2)

Unit – IV: Oscillation Theory
Three-term difference equations – Self-adjoint second order equations –Nonlinear difference equations. (Chapter 7: Sections: 7.1 to 7.3)

Unit – V: Asymptotic Behaviour of Difference Equations
Tools of approximations - Poincare's theorem – Asymptotically diagonal systems. (Chapter 8: Sections: 8.1 to 8.3)

Book for Study
Saber N.Elaydi, An Introduction to Difference Equations, Third Edition, Springer International Edition, First Indian Reprint, New Delhi, 2008.

Books for Reference

1.S.Goldberg, Introduction to Difference Equations, Dover Publications, 1986.
2.Walter G.Kelley, Allan C.Peterson, Difference Equations An Introduction with Applications, Academic Press, Indian Reprint, New Delhi, 2006.
3.V.Lakshmikantham, DonatoTrigiante, Theory of Difference Equations: Numerical Methods and Applications, Second Edition, Marcel Dekker, Inc, New York, 2002.
4.Ronald E.Mickens, Difference Equations, Van Nostrand Reinhold Company, New York, 1987.
5.Sudhir K.Pundir, Rimple Pundir, Difference Equations (UGC Model Curriculum), Pragati Prakashan, First Edition, Meerut, 2006.

E – Learning source:

MM 243 THEORY OF WAVELETS (Elective)

Text Book:
Michael Frazier*, An Introduction to Wavelets through Linear Algebra,* Springer

Prerequisites: Linear Algebra, Discrete Fourier Transforms, elementary Hilbert Space Theorems
(No questions from the pre-requisites)

UNIT I

Construction of Wavelets on ZN the first stage. (Section 3.1)

UNIT II

Construction of Wavelets on Zn the iteration sets, Examples - Shamon, Daubiehie and Haar
(Sections: 3.2 and 3.3)

UNIT III

ι2 (Z), Complete Orthonormal sets, L2[-π,π] and Fourier Series.
(Sections: 4.1,4.2 and 4.3)

UNIT IV

Fourier Transforms and convolution on ι2 (Z), First stage wavelets on Z.
(Section: 4.4 and 4.5)

UNIT V

The iteration step for wavelets on Z, Examples, Shamon Haar and Daubiehie

References:

Mayor (1993), *Wavelets and Operators,* Cambridge University Press
Chui. C(1992), *An Intrioduction to Wavelets,* Academic Press, Boston

MM 243 CODING THEORY(Elective)

Text: ***Coding Theory – An Introduction***, **San Ling and Chaoping Xing**

Cambridge Univ.Press, 2004.

Unit I

Introduction to Coding theory - Error Detection, Correction and Decoding - Basics of Finite Fields.

(Chapters 1, 2 and Sections 3.1, 3.2 , 3.3 of the Text)

Unit II

Linear Codes – Generator and Parity check matrices – Coding and decoding of linear codes.

(Chapter 4 of the Text)

Unit III

Some Bounds in Coding theory – Sphere Covering bound – Hamming bound and Perfect codes – Binary Hamming codes – Singleton bound and MDS codes.

(Sections 5.1, 5.2, 5.3.1, 5.4 of the Text)

Unit IV

Reed – Muller Codes – Cyclic codes – Generator and Parity check polynomials – Generator and parity check matrices – Decoding of Cyclic codes.

(Sections 6.2, 7.1, 7.2, 7.3, 7.4 of the Text)

Unit V

(A review of Section 3.3 of the Text is to be done as a prerequisite to this Unit)

BCH Codes – Decoding of BCH Codes – Reed-Solomon Codes

(Sections 8.1, 8.2 of the Text)

(Simple exercise problems of the corresponding sections are to be practiced).

References:

1. The Theory of Error Correcting Codes, F.J.MacWilliams and N.J.A.Sloane, North Holland, Amsterdam (1998).

2. Error Control Coding – Fundamentals and Applications, Shu Lin and Daniel J.Costello, Pearson Education India, 2011.

3.R. Lidl and H.Neiderreiter, Introduction to Finite Fields and their Applications, Cambridge University Press.

MM 243 ADVANCED ALGEBRA (Elective)

Text Book: David S. Dummit, Richard M.Foote, Abstract Algebra, 3rd Edition, Wiley 2011

UNIT I

Basic theory of field extenstions, Algebraic extenstions,

(Sections 13.1, 13.2)

UNIT II

Straight edge and compass constructions , Splitting fields and algebraic closures, Cyclotomic fields,

(Sections 13.3, 13.4)

UNIT III

Seperable and inseparable extensions, Existence and uniqueness of finite fields, (Section 13.5)

UNIT IV

Cyclotomic polynomials and extenstions, Basic definitions and examples related to fundamental theorem of Galoies theory

(Section 13.6, 14.1)

UNIT V

The fundamental theorem of Galois theory, Finite fields

(Sections, 14.1,14.2,14.3)

References

[1] Joseph Gallian, Contemporary Abstract Algebra, 8th Edition, Cengage Learning

[2] J B Fraleigh, A first course in Abstract Algebra, Seventh Edition, Pearson Eductation

[3] P B Bhattacharya, S K Jain, and S R Nagpaul, Basic Abstract Algebra, 2nd Edition, Cambridge University Press, 2004

[4] Michael Artin, Algebra, Prentice-Hall of India, 2003

MM 244 MECHANICS (Elective)

Text: Herbert Goldstein, Charles P. Poole and John Safko, *Classical Mechanics*, Third Edition, Pearson, 2011.

Unit I: Mechanics of a particle, Mechanics of a system of particles, Constraints, D'Alembert's principle and Lagrange's equations, Velocity dependent potentials and the dissipation function, Simple applications of the lagrangian formulation.(Chapter 1 of text)

Unit II: Hamilton's principle, Some techniques of the calculus of variations, derivation of Lagrange's equation from Hamilton's principle, Extending Hamilton's principle to systems with constraints, Conservation theorems and symmetry properties. (Sections 2.1, 2.2, 2.3, 2.4 and 2.6)

Unit III: Reduction to the equivalent one body problem, the equations of motion and first integrals, the equivalent one dimensional problem and classification of orbits, the Virial theorem, the differential equation for the orbits and integrable power law potentials, the Kepler problem: Inverse square law of force.(Sections 3.1, 3.2, 3.3, 3.4, 3.5 and 3.7)

Unit IV: The independent coordinates of a rigid body, orthogonal transformation, the Euler angles, the Cayley – Klein parameters and related quantities, Euler's theorem on the motion of a rigid body, the coriolis effect. (Sections 4.1, 4.2, 4.4, 4.5, 4.6, 4.10)

Unit V: Angular momentum and kinetic energy of motion about a point, tensors, the inertial tensor and the moment of inertia, the eigen values of the inertial tensor and the principal axis transformation, solving rigid body problems and the Euler equations of motion.

(Sections 5.1 to 5.5)

References:

Synge J.L and Griffith B.A, *Principles of Mechanics*, McGraw – Hill.

244 CRYPTOGRAPHY(Elective)

TEXT : **Douglas R. Stinson and Maura B. Paterson, Cryptography Theory and Practice, CRC Press, Taylor and Francis, 4th Edition. (2019)**

Unit 1

Cryptosystems and Basic Cryptographic Tools:,Secret-key Cryptosystems, Public-key Cryptosystems, Block and Stream Ciphers, Hybrid Cryptography, Message Integrity, Message Authentication Codes, Signature Schemes, Nonrepudiation, Certificates, Hash Functions, Cryptographic Protocols, Security

Classical Cryptography: Some Simple Cryptosystems, The Shift Cipher, The Substitution Cipher, The Affine Cipher, The Vigen`ere Cipher, The Hill Cipher, The Permutation Cipher , Stream Ciphers. [*Chapter 1 and chapter 2(section 2.1)*]

Unit 2

Cryptanalysis: Cryptanalysis of the Affine Cipher, Cryptanalysis of the Substitution Cipher, Cryptanalysis of the Vigen`ere Cipher, Cryptanalysis of the Hill Cipher, Cryptanalysis of the LFSR Stream Cipher

Shannons Theory: Elementary Probability Theory, Perfect Secrecy, Entropy, Huffman Encodings, Properties of Entropy, Spurious Keys and Unicity Distance,[*Chapter 2(section 2.2) and Chapter 3*]

Unit 3

Block Ciphers and Stream Ciphers: Introduction, Substitution-Permutation Networks, Linear Cryptanalysis, The Piling-up Lemma, Linear Approximations of S-boxes, A Linear Attack on an SPN , Differential Cryptanalysis , The Data Encryption Standard Description of DES , Analysis of DES, The Advanced Encryption Standard Description of AES , Analysis of AES , Modes of Operation, Padding Oracle Attack on CBC,Mode Stream Ciphers, Correlation Attack on a Combination Generator, Algebraic Attack on a Filter Generator,Trivium [Chapter 4]

Unit 4

Hash Functions and Message Authentication:Hash Functions and Data Integrity, Security of Hash Functions, The Random Oracle Model, Algorithms in the Random Oracle Model, Comparison of Security Criteria, Iterated Hash Functions, The Merkle-Damgard Construction, Some Examples of Iterated Hash Functions, The Sponge Construction, SHA-3 , Message Authentication Codes, Nested MACs and HMAC
, CBC-MAC, , Authenticated Encryption , Unconditionally Secure MACs , Strongly Universal Hash Families, Optimality of Deception Probabilities[]Chapter 5]

Unit 5

The RSA Cryptosystem and Factoring Integers: Introduction to Public-key Cryptography, More Number Theory, The Chinese Remainder Theorem , Other Useful Facts, The RSA Cryptosystem, Implementing RSA , Primality Testing, Legendre and Jacobi Symbols, The

Solovay-Strassen Algorithm, The Miller-Rabin Algorithm, Square Roots Modulo n, Factoring Algorithms, The Pollard p �1 Algorithm,The Pollard Rho Algorithm, Dixon's Random Squares Algorithm, Factoring Algorithms in Practice, Other Attacks on RSA, The Decryption Exponent, Wiener's Low Decryption Exponent Attack, The Rabin Cryptosystem , Security of the Rabin Cryptosystem, Semantic Security of RSA , Partial Information Concerning Plaintext Bits, Obtaining Semantic Security[Chapter 6]

References

6.Jeffrey Hoffstein: Jill Pipher, Joseph H. Silverman, An Introduction to Mathematical Cryptography, Springer International Edition.

7.H. Deffs & H. Knebl: Introduction to Cryptography, Springer Verlag, 2002.

8.Alfred J. Menezes, Paul C. van Oorschot and Scott A. Vanstone: Handbook of Applied Cryptography, CRC Press, 1996.

9.William Stallings: Cryptography and Network Security Principles and Practice, Third Edition, Prentice-hall India, 2003.

MM 244 ADVANCED GRAPH THEORY (Elective)

Text:

Fred Buckley, Frank Harary, *Distance in Graphs,* Addison-Wesley Publishing Company

UNIT 1

Graphs: Graphs as Models, Paths and connectedness, Cutnodes and Blocks, Graph Classes and Graph Operations, Polynomial Algorithms and NP-Completeness
(Chapter 1 and Section 11.1 of Text)

UNIT II

The Center and Eccentricity, Self Centered Graphs, The Median, Central Paths Path Algorithms and Spanning Trees, Centers.
(Chapter 2, Sections 2.1, 2.2, 2.2; Chapter 11, Sections 11.2, 11.3)

UNIT III

External Distance Problems: Radius, Small Diameter, Diameter, Long Paths and Long Cycles
(Chapter 5 of Text)

UNIT IV

Convexity: Closure in variants, Metrics on Graphs, Geodetic Graphs, Distance Hereditary Graphs.
Diagraphs: Diagraphs and Connectedness, Acyclic diagraphs
(Chapter 7 and sections 10.1, 10.2 of Text)

UNIT V

Distance Sequences: The eccentric sequences, Distance sequence, The Distance distribution, Long Paths in Diagraphs, Tournaments
(Sections 9.1, 9.2,9.3,10.3,10.4 of Text)

References:

(1) **Bondy and Murthy, *Graph Theory with Applications,* The Macmillan Press Limited, 1976**
(2) **Chartrand G and L.Lesniak, *Graphs and Diagraphs,* Prindle, Weber and Schmidt, Boston, 1986**
(3) **Garey M.R, D.S Johnson , *Computers and Intractability*, A Guide to the Theory of NP-Completeness, Freeman, San Francisco 1979.**
(4) **Harary. F, *Graph Theory,* Addison Wesley Reading Mass 1969 (Indian Edition, Narosa)**
(5) **K.R Parthasarathy, *Basic Graph Theory,* Tata Mc Graw-Hill, Publishing Co, New Delhi, 1994.**

MM 244 COMMUTATIVE ALGEBRA (Elective)

Text: N.S Gopalakrishan, *Commutative Algebra,* Oxonian Press

UNIT I

Modules, Free projective, Tenser product of modules, Flat modules
(Chapter 1 of Text)

UNIT II

Ideals, Local rings, Localization and applications
(Chapter 2 of Text)

UNIT III

Noetherian rings, modules, Primary decomposition, Artinian modules
(Chapter 3 of Text)

UNIT IV

Integral domains, Integral extensions, Integrally closed domain, Finiteness of integral closure
(Chapter 4 of Text)

UNIT V

Valuation rings, Dedikind domain
(Chapter 5 of Text, Theorems 4 and 5 omitted)

References:

[1] M.F Atiyah and I.G Mac Donald, *Introduction to Communication Algebra,* Addison Wesley
[2] T.W Hungerford, *Algebra,* Springer-Verlag

MM 244: ADVANCED COMPLEX ANALYSIS (Elective)

Text: John. B. Conway, Functions of Complex Variables, Springer-Verlag, New York, 1973. (Indian Edition: Narosa)

UNIT I

Compactness and Convergence in the space of Analytic functions, The space C(G,Ω), Space of Analytic functions, Riemann Mapping Theorem.

(Chapter 7- Sections 1, 2 and 4 of the Text)

UNIT II

Wierstrass factorization Theorem, Factorization of sin function, The Gamma function.

(Chapter 7- Sections 5,6 and 7 of the Text)

UNIT III

Riemann Zeta function, Runge's Theorem, Simple connectedness, Mittag-Leffler's Theorem.

(Chapter 7- Section 8 and Chapter 8 of the Text)

UNIT IV

Analytic continuation and Riemann surfaces, Schwarz Reflexion Principle, Analytic continuation along a path, Monodromy Theorem.

(Chapter 9- Sections 1, 2 and 3 of the Text)

UNIT V

Basic properties of Harmonic functions, Harmonic function on a disc, Jensen's formula, The genus and order of an entire function, Hadamard factorization Theorem.

(Chapter 10- Sections 1, 2 and Chapter 11 of the Text)

References:

1. L.V. Ahlfors, Complex Analysis, Mc-Graw Hill (1966)

2. S. Lang, Complex Analysis, Mc-Graw Hill (1998).

3. S. Ponnusamy & H. Silverman, Complex Variables with Applications, Birkhauser.

4. H. A. Priestley, Introduction to Complex Analysis, Oxford University Press Tristan Needham, Visual Complex Analysis, Oxford University Press(1999)

5. V. Karunakaran, Complex Analysis, Narosa Publishing House,

MM 244 REPRESENATION THEORY OF FINITE GROUPS
(Elective)

Text: Walter Ledermann, ***Introduction to Group Characters,*** **Cambridge University Press**

UNIT I

G-module, Characters, Reducibility, Permutation representations, Complete reducibility, Schur's Lemma
(Sections 1.1 to 1.7 of Text)

UNIT II

The commutant algebra, Orthogonality relations, The groups algebra
(Section 1.8, 2.1, 2.2 of Text)

UNIT III

Character table, Character of finite abelian groups, The lifting process, Linear characters
(Section 2.3, 2.4, 2.5, 2.6 of Text)

UNIT IV

Induced representations, Reciprocity law, A_5, Normal subgroups, Transitive groups, Induced characters of S_n
(Sections 3.1, 3.2, 3.3, 3.4, 4.1, 4.2, 4.3 of Text)

UNIT V

Group theoretical applications, Brunside's (p,q) Theorem, Frobenius groups
(Chapter 5 of Text)

Reference: S.Lang, ***Algebra,*** **Addison Wesley**

MM 244 CATEGORY THEORY (Elective)

Text Book: S. Maclane, *Categories for the working Mathematician,* Springer, 1971

UNIT-I

Categories, Functors and Natural Transformations - Axioms for categories, categories, Functors. Natural Transformations, Mobics, Epis and Zeros Foundations, Large Categories, Hom-sets.

UNIT II

Constructions on categories - Duality Contravariance and opposites, Products of Categories. Functor Categories, The category of all categories, Comma categories, Graphs and Free categories, Quotient Categories.

UNIT III

Universals and Limits - Universal Arrows, Yoneda Lemma Coproduces and Colimits, Products and Limits, Categ ories with Finite products, Groups in categories.

UNIT IV

Adjoints – Adjunctions, Examples of Adjoints, Reflective subcategories, Equivalence of categories, Adjoints for pre orders, Cartesian closed categories, Transformations of Adjoints, Compositions of Adjoints.

UNIT –V

Limits – Creation of Limits by products and Equalizers, Limits with parameters, Preservation of Limits, Adjoints on Limits, Freyd's Adjoint Functor Theorem, Subobjects and Generation, The Special Adjoint Functor Theorem, Adjoint in Topology.

References:

1. **M.A. Arbib and E.G Maneswarrows, *Structures and Functors,* The categorical Imperative, Avademic Press-1975**
2. **H. Herrlich and G.E Strecker, *Category Theory,* Allyn & Bacon, 1973**
3. **M. Barmand, C. Wells, *Category Theory for Computer Science,* Prentice Hall , 1990**
4. **F. Borceux, *Handbook of Categorical Algebra,* Vol. I, II, III, Cambridge, University Press, 1994**
5. **P. Frevd, *Abelian Categories,* Harper & Row, 1964**
6. **R.F,C Walters, *Categories and Computer Science,* Cambridge University Press, 1991**

MM 244 SPECTRAL GRAPH THEORY

Text: Bogdan Nica, A Brief Introduction to Spectral Graph Theory, European Mathematical Society Publishing House, 2018.

Unit I

A quick review of chapter I, Invariants - Chromatic number and independence number (Section 2.1 of Chapter II), Eigenvalues of graphs – Adjacency and Laplacian eigenvalues, First properties, First examples (Chapter 7).

Unit II

Eigenvalue computations - Cayley graphs and bi-Cayley graphs of abelian groups, Strongly regular graphs, Two gems, Design graphs (Chapter 8) (Except Example 8.10 and Example 8.23).

Unit III

Largest eigenvalues - Extremal eigenvalues of symmetric matrices, Largest adjacency eigenvalue, The average degree, A spectral Turán theorem, Largest Laplacian eigenvalue of bipartite graphs, Sub graphs, Largest eigenvalues of trees (Chapter 9).

Unit IV

More eigenvalues - Eigenvalues of symmetric matrices: Courant–Fischer, A bound for the Laplacian eigenvalues, Eigenvalues of symmetric matrices: Cauchy and Weyl, Sub graphs (Chapter 10).

Unit V

Spectral bounds - Chromatic number and independence number, Isoperimetric constant, Edge counting (Chapter 11).

References:

1. D. Cvetkovic, M. Doob, H. Sachs, Spectra of Graphs - Theory and Application, Acadamic Press, New York, 1980.

2. Andries E. Brouwer, William H. Haemers, Spectra of graphs – Monograph, Springer.2011.