UNIVERSITY OF KERALA
M.PHIL (STATISTICS) ENTRANCE EXAMINATION- 2019
(MODEL QUESTION PAPER)

Time: 3 Hours **Total Marks: 100**

Part-I

*Answer **all** questions. Each question carries **one** mark.*

1. If X and Y are two random variables the covariance between the variables aX+b and cY+d in terms of Cov(X, Y) is
   (a) Cov(X, Y) (b) abcd COV(X, Y) (c) ac Cov(X, Y) (d) Cov(X, Y) + bd
2. If X is distributed as b (n, p) the distribution of Y = n-X is
   (a) b(n, 1) (b) b(n, x) (c) b(n, p) (d) b(n,1-p)

3. If (X, Y) follows bivariate N(0,0,1,1, ρ), then the variables X+Y and X-Y are
   (a) Correlated with ρ=1/2 (b) independently distributed (c)Negatively correlated (d) none of the above
4. A Poisson random variable has $\mu_4 = 2$. The value of its variance is
   (a) 1/3 (b) 2/3 (c) 4/9 (d) 1/9
5. Let X and Y be two independent Gamma variates with parameters α_1, β and α_2, β respectively. Then the random variable X/Y has
   (a) Gamma distribution (b) Beta distribution of first kind (c) Beta distribution of second kind (d) Chi-square distribution
6. Let $X \sim N_p(\mu, \Sigma)$ and let A be a p×p matrix of constants. Then $X'AX \sim \chi_r^2$ if
   (a) A is an idempotent matrix of rank r
   (b) Σ^{-1} is an idempotent matrix of rank r
   (c) $A\Sigma$ is an idempotent matrix of rank r
   (d) $A\Sigma^{-1}$ is an idempotent matrix of rank r
7. If $E(X) = E(X^2) = 0$, the value of P(X=0) is

   (a) 0 (b) 0.25 (c) 0.5 (d) 1

8. If X is a random variable such that $P(|X| \leq k) = 1$, for some $k<\infty$, then X has
   (a) finite expectation (b) finite variance (c) finite moments up to order k (d) finite moments of any order
9. Let X be a random variable with $0 \leq \alpha_n = E(|X|^n) < \infty$, for $n \geq 1$. Then the sequence $\{\alpha_n^{\frac{1}{n}}, n \geq 1\}$ is
   (a) decreasing (b) increasing (c) non-decreasing (d) non- increasing
10. $X_n \xrightarrow{d} X \Rightarrow X_n \xrightarrow{P} X$ if
    (a) X_n and X are independent and identically distributed random variables
    (b) X_n and X are normally distributed random variables
    (c) X is a degenerate random variable
    (d) X is a uniform random variable

11. The sequence $\{X_n, n \geq 1\}$ of independent and identically distributed random variables obeys WLLN if (a) $E(X_n) < \infty$ (b) $Var(X_n) < \infty$ (c) $E|X_n| < \infty$ (d) $\sum_{k=1}^{\infty} \frac{Var(X_k)}{k^2} < \infty$

12. Let $\{X(t), t \geq 0\}$ be a stochastic process with stationary independent increments and let X(0) = 0 and Var{X(1)}= σ^2. Then Cov{X(t), X(s)} is
(a) 0 (b) σ^2ts (c) σ^2 min {t, s} (d) σ^2 max{t, s}

13. Consider a Markov Chain with state space $\{0,1\}$ and transition probability matrix $P = \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.
Then
(a) State 1 is recurrent and state 0 is transient (b) State 0 is recurrent and state 1 is transient
(c) Both states are recurrent (d) Both states are transient

14. Let the distribution of the number of offsprings be geometric with $p_k = q^k p,\ k = 0,1,2,\ldots,\ 0 < p < 1$ and $q = 1 - p$. Then the probability of extinction is certain if and only
(a) $q \leq p$ (b) $q > p$ (c) $q = p$ (d) $q \neq p$

15. Let be a Poisson variate with parameter λ. An unbiased estimator of $e^{-\lambda}$ is the indicator function
T(X) = 1 if X = 0
= 0 if X ≥ 1
Which of the following statements is true about T(X)?
(a) Variance of T(X) attains the Cramer-Rao lower bound
(b) Variance of T(X) is less than the Cramer-Rao lower bound
(c) T(X) is U.M.V.U.E. of $e^{-\lambda}$
(d) T(X) is not a U.M.V.U.E. of $e^{-\lambda}$

16. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with pd.f.
$f(x;\theta) = e^{-(x-\theta)}, x > \theta, \theta > 0$
= 0 elsewhere
A sufficient statistic for θ is
(a) Min $(X_1, X_2, \ldots, X_n)$ (b) Max $(X_1, X_2, \ldots, X_n)$ (c) Median of $(X_1, X_2, \ldots, X_n)$ (d) Mean of $(X_1, X_2, \ldots, X_n)$.

17. Let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of a random sample of size n from a distribution with p.d.f.
$f(x:\theta) = \frac{1}{\theta}, \quad 0 < x < \theta, \theta > 0$
= 0 elsewhere
The statistic Y_n is
(a) Complete but not sufficient (b) not complete but sufficient (c) not complete and not sufficient (d) complete and sufficient.

18. Let X be a binomial variate with parameter n and p, where ϕ has uniform prior distribution over (0, 1). Then the Bayes estimator of p w.r.to squared error loss function is
(a) $\frac{X+1}{n+2}$ (b) $\frac{X+1}{n+1}$ (c) $\frac{X}{n}$ (d) none of the above

19. A size α test φ of $H_0 : \theta \in \theta_0$ against the alternative $H_1: \theta \in \theta_1$ is unbiased. Then its power function satisfies:

(a) $\beta_\varphi(\theta) \leq \alpha$ for $\theta \in \theta_0$ and $\beta_\varphi(\theta) \geq \alpha$ for $\theta \in \theta_1$
(b) $\beta_\varphi(\theta) \leq \alpha$ for both $\theta \in \theta_0$ and $\theta \in \theta_1$
(c) $\beta_\varphi(\theta) \geq \alpha$ for $\theta \in \theta_0$ and $\beta_\varphi(\theta) \leq \alpha$ for $\theta \in \theta_1$
(d) $\beta_\varphi(\theta) \geq \alpha$ for both $\theta \in \theta_0$ and $\theta \in \theta_1$

20. It is proposed to test H_0: $\theta=2$ against H_1: $\theta=1$ on the basis of the single observation X from the distribution $f(x/\theta) = \theta\, e^{-\theta x}$, $x \geq 0$. The test procedure is to reject H_0 if $X \leq 1$. The size S and power P of the test procedure are:
(a) $S= e^{-2}, P=e^{-1}$ (b) $S=1-e^{-2}, P=e^{-1}$ (c) $S=1-e^{-2}, P=1-e^{-1}$ (d) $S=e^{-2}, P=1-e^{-1}$

21. Under proportional allocation in stratified sampling the size of the sample from each stratum depends on
(a) total sample size (b) size of the stratum (c) population size (d) all the above

22. The probability of selecting a sample is proportional to sum of measures of sizes of the units included in the sample if the sampling is done according to
(a) SRSWOR (b) PPSWR (c) Midzuno-Sen scheme (d) PPSWOR

23. In the following which sampling strategy is unbiased for estimating population mean?
(a) (SRSWOR, Ratio estimator) (b) (PPSWR, ratio estimator) (c) (SRSWOR, Regression estimator) (d) (PPSWOR, Regression estimator)

24. Circular systematic sampling was first used by
(a) W.G. Cochran (b) M. H. Hansen (c) D. B. Lahiri (d) P.C. Mahalanobis

25. In a Latin square design with five treatments and with two missing plots, what is the number of error degrees of freedom?
(a) 10 (b) 11 (c) 12 (d) 16

26. In a completely randomized design there are 5 treatments A, B, C, D and E. The treatments A,B,C,D are replicated 3, 4, 5, and 6 times respectively. If in the ANOVA table the degrees of freedom for error is 19, then E is replicated:
(a) 4 times (b) 5 times (c) 6 times (d) 7 times

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29. For a 3^2 factorial experiment arranged in 3 blocks, the number of confounded interactions is
(a) 1 (b) 2 (c) 3 (d) 6

30. While writing research report a researcher

(a) must not use the numerical figures in numbers in the beginning of sentences
(b) must arrange it in logical, topical and chronological order
(c) must compare his results with those of the other studies

(d) all of the above

31. The per capita income of India from 1950 to 1990 is four times. This study is
(a) social

(b) horizontal
(c) longitudinal
(d) factorial

32. If you are doing experiment on a large group of sample which method of controlling will you adopt?

(a) matching
(b) randomization
(c) elimination and matching both
(d) elimination

33. A good hypothesis should be

(a) precise, specific and consistent with moist known facts
(b) formulated in such a way that it can be tested by the data
(c) of limited scope and should not have global significance
(d) all of these

34. All causes non sampling errors except

(a) faulty tools of measurement
(b) inadequate sample
(c) non response
(d) defect in data collection

35. A researcher wants to study the future of the Congress in India. For the study which tool is most appropriate for him?

(a) Questionnaire
(b) Schedule
(c) Interview
(d) Rating scale

36. The other name of independent variable for an experimental research is/are

(a) treatment variable
(b) experimental variable
(c) exogenous variable
(d) all of the above

37. Bibliography given in a research report

(a) helps those interested in further research and studying the problem from another angle

(b) makes the report authentic
(c) Both (a) & (b)
(d) none of the above

38. If the sample drawn does not specify any condition about the parameter of the population, it is called

(a) selected statistics
(b) distribution free statistics
(c) census
(d) none of the above

39. Area (cluster) sampling technique is used when

(a) population is scattered and large size of the sample is to be drawn
(b) population is heterogeneous
(c) long survey is needed
(d) (a) and (c)

40. Validity of a research can be improved by

(a) eliminating extraneous factors
(b) taking the true representative sample of the population
(c) both of the above measures
(d) none of these

Part II

*Answer any **ten** questions. Each question carries **six** marks.*

41. Given $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}$. Determine the characteristic roots of A. Hence prove or disprove $A^4 = 6A^3-11A^2+6I$, where I is the unit matrix of order 3.

42. Let X be a continuous type random variable with p. d. f. f(x), whose graph is symmetric with respect to x=c. If the mean value exists, show that E(X) = c.

43. Let X_1, X_2, X_3, X_4, X_5 be five mutually stochastically independent random variables each with p. d. f

$$f_X(x) = \begin{cases} 3(1-x^2), 0 < x < 1 \\ 0, \textit{eisewhere} \end{cases}$$

If Y is the minimum of these variables, find the distribution function and p. d. f. of Y.

44. Let $X = (X, Y)'$ follows $N_2(0, \Sigma)$ where $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$. Find the distribution of (X+Y, X-Y).

45. If $X_1, X_2, \ldots, X_n$ are independent identically distributed standard normal variates, then show that $\frac{1}{n}\left(X_1^2 + X_2^2 + \ldots + X_n^2\right) \xrightarrow{P} 1$ as $n \to \infty$.

46. Examine whether the random process {X(t)=10cos (100 t +U), t ε T} is a wide sense stationary process if (i) U is uniformly distributed over (0,π) and (ii) U is uniformly distributed over (0,2π).

47. Let $X = (X_1, X_2, \ldots, X_n)$ be a sample from $N(\alpha\sigma, \sigma^2)$ where α is a known real number. Show that the statistic $T(X) = \left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$ is sufficient for σ^2.

48. A random sample of size n is taken from a normal population with mean μ and variance unity. Does there exist a UMP level α test for testing H_0: $\mu \leq 2$ against H_1 : $\mu > 2$? If so why and what is it?

49. How do you define a research problem? Discuss about the objectives of research.

50. Give an account of various types of research.

51. Explain the basic principles of experimental designs.

52. Explain different types of reports, particularly pointing out the difference between a technical report and a popular report.

53. Describe various types of illustrations used in research reports.

54. Explain different types of intellectual property rights.

55. Explain various types of sampling methods.