MODEL QUESTION FOR FIFTH SEMESTER B.Sc MATHEMTICS

(2014 Admission)

MM 1544- Vector Analysis

Time: 3 Hours Max. mark: 80

SECTION- 1

All the ten questions are compulsory. They carry one mark each

1. Find the gradient of \( \phi(x, y) = x + y \).
2. State Green’s Theorem
3. Find the directional derivative of \( f(x, y) = e^{xy} \) at (-2,0)
4. S.T div(curl F) = 0
5. State the Conservation of Energy principle
6. Find the slope of the surface \( Z = xy \) in the direction of the vector \( u = i + j \) at (1,2,2)
7. When we say that a vector field is conservative
8. Evaluate \( \int_C ds \), if \( C \) is the line segment from (0,0) to (1,0)
9. State the fundamental theorem of work integrals
10. Given that \( r = xi + yj + zk \). show that curl(\( r \)) = 0

SECTION -2

Answer any eight from the following. Each question carries 2 marks

11) Use line integral to find the area enclosed by the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \)
12) Show that the divergence of the inverse square field \( F(x, y, z) = \frac{c}{\|r\|^3} \) is zero
13) Evaluate \( \int_C (1 + xy^2)ds \) from (0,0) to (1,2) along the line segment that is represented by

the parametric curve \( x = t, y = 2t, 0 \leq t \leq 1 \)
14) let \( F(x, y) = 2xy^3i + (1 + 3x^2y^2)j = \nabla \phi \), Find \( \phi \)
15) use Green’s Theorem t t evaluate \( \int_C x^2y dx + xdy \) along the circular path joined the points (0,0), (1,0), and (1,2)
16) Use the divergence theorem to find the outward flux of the vector field \( F(x,y,z) = z \mathbf{k} \) across the sphere \( x^2 + y^2 + z^2 = a^2 \)

17) Find \( \text{curl(curl } F) \) for the function \( F(x,y,z) = y^2 x \mathbf{i} - 3xyz \mathbf{j} + xy \mathbf{k} \)

18) If \( f \) and \( g \) are differentiable functions, show that \( \nabla \left( \frac{f}{g} \right) = \frac{g \nabla f - f \nabla g}{g^2} \)

19) Find the directional derivative of \( f(x,y) = e^{-x} \cos y \) at \( (0, \frac{\pi}{4}) \)

20) Determine whether the vector field \( F(x,y,z) = (x + y + z) \mathbf{i} + (y - x - z) \mathbf{j} + 3 \mathbf{k} \) is conservative on some open set

21) Define an "Inverse Square Field". State Gauss Law for Inverse Square Fields

22) Find the work done by the force field \( F(x,y) = x^2 \mathbf{i} + y^2 \mathbf{j} \) along the parabola \( y = x^2 \) from \((0,0)\) to \((1,1)\)

**SECTION- 3**

Answer any six from the following. Each question carries 4 marks

23) Evaluate \( \int_C \frac{-ydx + xdy}{x^2 + y^2} \), where \( C \) is a piece wise smooth closed curve oriented counter clockwise such that \( C \) encloses the origin

24) Let \( \sigma \) be the portion of the surface \( z = 1 - x^2 - y^2 \) that lies above the \( xy \)-plane and suppose \( \sigma \) is oriented up. Find the flux of the vector field \( F(x,y,z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \) across \( \sigma \)

25) The temperature at a point \((x,y,z)\) in a metal sheet is \( T(x,y,z) = \frac{xyz}{1 + x^2 + y^2 + z^2} \).

Find the rate of change of temperature with respect to distance at \((1,1,1)\) in the direction of the origin

26) Find the mass of a thin wire shaped in the form of a circular arc \( y = \sqrt{9 - x^2} \), \( 0 \leq x \leq 3 \) if the density function is \( \delta(x,y) = x \sqrt{y} \)

27) Show that the line integral \( \int_C y \sin x \, dx - \cos x \, dy \) is independent of the path and hence evaluate \( \int_{(0,1)}^{(\pi,1)} y \sin x \, dx - \cos y \, dy \)
28) Using Green’s theorem find the work done by the force field
\[ F(x, y) = (e^y - y^3)i + (\cos y + x^3)j \]

29) Suppose that a curved lamina \( \sigma \) with constant density \( \delta(x, y, z) = 1 \) is the portion of the paraboloid \( z = x^2 + y^2 \) below the plane \( z = 1 \). Find the mass of the lamina.

30) Use divergence theorem to find the outward flux of the vector field
\[ F(x, y, z) = 2xi + 3yj + z^2k \] across the unit cube bounded by the planes \( x = 0, x = 1, y = 0, y = 1, z = 0, \) and \( z = 1 \).

31) Using Stoke’s Theorem, evaluate \( \int_C F \cdot dr \), where \( F(x, y, z) = z^2i + 2xj - y^3k \), \( C \) is the circle \( x^2 + y^2 = 1 \) in the xy-plane with counter clockwise orientation looking down the z-axis.

SECTION -4

Answer any two from the following. Each question carries 15 marks

32) Prove that
   a) \( \text{div}(\phi F) = \phi \text{div}F + \nabla \phi \cdot F \)
   b) \( \text{curl}(\phi F) = \phi \text{curl}F + \nabla \phi \times F \)

33) Verify divergence theorem for the function \( F(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \), where \( \sigma \) is the spherical surface \( x^2 + y^2 + z^2 = 1 \)

34)
   a) Let \( T(x, y) = 10 - 8x^2 - 2y^2 \). Find the maximum value of a directional derivative at (2,3). Also find the unit vector in the direction in which the maximum value occurs.
   b) A heat seeking particle is located at the point (2,3) in a flat metal plate whose temperature at a point \( (x, y) \) is \( T(x, y) = 10 - 8x^2 - 2y^2 \). Find an equation of the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

35) Verify Stoke’s Theorem for the vector field \( F(x, y, z) = 2zi + 2xj + 5yk \) taking \( \sigma \) to be the portion of the paraboloid \( z = 4 - x^2 - y^2 \) for which \( z \geq 0 \) with upward orientation, and \( C \) to be positively oriented circle \( x^2 + y^2 = 4 \) that forms the boundary of \( \sigma \) in the xy-plane.