University of Kerala  
First degree programme in Mathematics  
Fifth Semester B.Sc. Degree model questions  
COMPLEX ANALYSIS  
MM 1542  
(2014 Admissions onwards)  

Time: 3 Hours  
Max. Marks: 80

All the first 10 questions are compulsory. They carry 1 mark each.

1. Express \( \frac{(4-i)(3+i)}{(2-i)} \) in the form \( a + ib \)

2. Find the square root of \( 4i \)

3. Show that \( |z|^2 = z \bar{z} \).

4. Find the radius of convergence of the series \( \sum_{n=0}^{\infty} z^n \).

5. If \( z = x + iy \), find \( |e^z| \)


7. Express \( 3 + 2i \) in polar form.

8. Write an example for a Cauchy sequence in Complex plane.

9. Let \( \{z : z = \bar{z} \} \). What is it geometrically means?

10. Write the power series representation of \( e^z \).

(10x1=10 Marks)
Answer any 8 questions from among the questions 11 to 22. These questions carry 2 marks each.

11. Find the sum of complex numbers $3 + 2i$ and $1 + i$ geometrically.

12. Find the cube roots of unity.

13. Prove that $\{z_n\}$ converges if and only if $\{z_n\}$ is a Cauchy Sequence.

14. Using Cauchy Riemann equation, verify $-x^2 + y^2 - 2xy i$ is analytic.

15. Find the radius of convergence of. Is the series $\sum (1/2)^n$ converge or diverge. Justify your answer.

16. Prove that $f$ is constant if $f = u + iv$ is analytic in a region D and $u$ is constant.

17. Let $f(z) = \frac{1}{z}$ and $C: z(t) = R \cos t + i R \sin t, 0 \leq t \leq 2\pi, R \neq 0$. Then find $\int_C f(z)dz$.

18. Evaluate $\int_C f(z)dz$ where $f(z) = x^2 + y^2$ and $C$ is $z(t) = t^2 + i t^2$

$0 \leq t \leq 1$.

19. Solve $x^3 + 4x + 2$ by Cubic Method.

20. Is the polynomial $x^2 + y^2 - 2xy i$ is analytic. Justify your answer.

21. Suppose $C$ is given by $z(t), a \leq t \leq b$. Then prove that

$$\int_C f = \int_C f.$$ 

22. Let $f(z) = |z|^2$. Is $f$ differentiable at $z = 0$. Justify.

(8x2=16 Marks)

Answer any 6 questions from the questions 23 to 31. These questions carry 4 marks each.

23. Geometrically represent the following sets.
24. Prove that \( |z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2) \) and interpret the Result geometrically.

25. Prove that if a polynomial \( P(x, y) \) is analytic if and only if \( P_x = i P_y \).

26. State and prove uniqueness theorem for power series.

27. Suppose \( f \) is derivative of an analytic function \( F \) - that is \( f(z) = F'(z) \)

Where \( F \) is analytic on the smooth curve \( C \).

Then \( \int_C f(z)dz = F(z(b)) - F(z(a)) \).

28. a) Evaluate \( \int_C (z - i)dz \) where \( c \) is the parabolic segment \( z(t) = t + i t^2 \)

\[-1 \leq t \leq 1.\]

b) Also find the above integral along the straight line \(-1 + i \) to \( 1 + i \).

29. Let \( C \) be a smooth curve; let \( f \) and \( g \) be continuous function on \( C \); and

Let \( \alpha \) be any complex number. Then

a) \( \int_C (f(z) + g(z))dz = \int_C (f(z))dz + \int_C (g(z))dz \).

b) \( \int_C \alpha f(z)dz = \alpha \int_C f(z) \).

30. Verify the following identities

a) \( \sin 2z = 2 \sin z \cos z \).

b) \( \sin^2z + \cos^2z = 1.\)

31. Is the following polynomials are analytic. Verify

a) \( P(x, y) = x^3 - 3xy^2 - x + i (3x^2 - y^3 - y) \)

b) \( P(x, y) = 2xy + i (y^2 - x^2). \) (6x4=24 marks)
Answer any 2 questions from among the questions 32 to 35. These questions carry 15 marks each.

32. a) Suppose that \( f(z) = \sum_{n=0}^{\infty} c_n z^n \) converges for \( |z| < R \). Then \( f'(z) \) exists and equals \( \sum_{n=0}^{\infty} n c_n z^{n-1} \) throughout \( |z| < R \).
   
b) The power series are infinitely differentiable within their domain of convergence.

33. a) If \( f = u + iv \) is differentiable at \( z \), \( f_x \) and \( f_y \) exists there and satisfy the Cauchy Riemann equation \( f_y = i f_x \).
   
b) Is the converse of the above statement is true. Justify your answer.
   
c) Show that \( f(z) = \text{Re} z \) is nowhere differentiable.

34. a) Suppose \( f \) is entire and \( \Gamma \) is the boundary of a rectangle \( R \).
   
   Then \( \int_{\Gamma} f(z) \, dz = 0 \).

   b) State and prove integral theorem.

35. a) Suppose \( G(t) \) is a continuous complex-valued function of \( t \). Then
   
   \[ \int_{a}^{b} G(t) \, dt \ll \int_{a}^{b} |G(t)| \, dt. \]

   b) Suppose that \( C \) is a smooth curve of length \( L \), that \( f \) is continuous on \( C \), and that \( f \ll M \) throughout \( C \). Then \( \int_{C} f(z) \ll ML \).

\( (15 \times 2 = 30) \)