UNIVERSITY OF KERALA  
Model Question Paper  
First Degree Programme in Statistics  
Semester IV  
MM 1431.4 Mathematics – IV  
(Linear Algebra)  

Time: 3 hours  
Maximum Marks: 80  

Section-I  
All the first 10 questions are compulsory. They carry 1 mark each.

1. Are the following matrices row equivalent:  
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 5 & 3 & 7 \\
0 & 4 & 9 & 10 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
and
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 3/5 & 7/5 \\
0 & 4 & 9 & 10 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

2. Check whether the matrix  
\[
\begin{bmatrix}
1 & 1 & -3 \\
2 & 16 & 1 \\
0 & 0 & 4 \\
\end{bmatrix}
\]
is singular.

3. Find the inverse of the matrix:  
\[
\begin{bmatrix}
3 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2 \\
\end{bmatrix}
\]

4. Find the rank of the matrix:  
\[
\begin{bmatrix}
1 & 3 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

5. Define a dilation from \(\mathbb{R}^2\) to \(\mathbb{R}^2\).

6. Write down the standard matrix corresponding to the transformation of reflection in the line  
\(x_2 = -x_1\).

7. State true or false: If \(A\) is an \(m \times n\) matrix and the transformation \(x \rightarrow Ax\) is onto, then \(\text{rank } A = m\).

8. Let \(A\) be a \(7 \times 5\) matrix. What must \(m\) and \(n\) be in order to define \(T: \mathbb{R}^m \rightarrow \mathbb{R}^n\) by \(T(x) = Ax\)

9. If a vector space \(V\) has a basis of \(n\) vectors, then every basis of \(V\) must consist of exactly \(\ldots\) vectors.

10. Find the matrix of the quadratic form: \(Q(x) = x_3^2 - 4x_1x_2 + 4x_1x_3\)

Section-II  
Answer any 8 questions from among the questions 11 to 22.  
These questions carry 2 marks each.

11. If \(A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}\), find \(A^n\)

12. If \(A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\), show that \(A^2 - 4A + 5I = 0\)

13. Find the Eigen values of the matrix:  
\[
\begin{bmatrix}
1 & 3 \\
2 & 1 \\
\end{bmatrix}
\]
14. By reducing it to the echelon form, find the rank of the matrix:
   \[
   \begin{bmatrix}
   1 & 1 & -1 \\
   1 & 2 & 1 \\
   -1 & 1 & 3
   \end{bmatrix}
   \]

15. Find the index of the nilpotent matrix:
   \[
   \begin{bmatrix}
   0 & 0 & 0 \\
   2 & 0 & 0 \\
   0 & -2 & 0
   \end{bmatrix}
   \]

16. Show that a matrix and its transpose have the same Eigen values.

17. Is \((1, -2)\) and \((-2, 4)\) a basis for \(\mathbb{R}^2\) over \(\mathbb{R}\)?

18. Define a linear transformation and check whether the transformation \(T\) is linear if \(T\) is defined by:
   \[T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2).\]

19. Let \(T\) be the linear transformation defined by
   \[
   T(e_1) = (1, 4), \quad T(e_2) = (-2, 9) \quad \text{and} \quad T(e_3) = (3, -8),
   \]
   where \(e_1, e_2\) and \(e_3\) are columns of the \(3 \times 3\) identity matrix. Check whether \(T\) is one-one or not.

20. Let \(v_1 = [3 \quad 6 \quad 2]^T, \quad v_2 = [-1 \quad 0 \quad 1]^T, \quad x = [3 \quad 12 \quad 7]^T\) and \(B = \{v_1, v_2\}\). Find the co-ordinate vector \([x]_B\) of \(x\) relative to \(B\).

21. Find the dimension of the null space and the column space of:
   \[
   \begin{pmatrix}
   1 & 3 & -4 & 2 & -1 & 6 \\
   0 & 0 & 1 & -3 & 7 & 0 \\
   0 & 0 & 0 & 1 & 4 & -3 \\
   0 & 0 & 0 & 0 & 0 & 0
   \end{pmatrix}
   \]

22. If \(A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\), show that \(A^3 - 6A^2 + 11A - 6I = 0\)

Section-III

Answer any 6 questions from among the questions 23 to 31.
These questions carry 4 marks each.

23. Test for consistency and solve:
   \[5x + 3y + 7z = 4; \quad 3x + 26y + 2z = 9; \quad 7x + 2y + 10z = 5\]

24. Let \(A\) be an \(n \times n\) matrix with \(n\) distinct Eigen values. Prove that the corresponding Eigen vectors are linearly independent.

25. Find the Eigen vectors of the matrix:
   \[
   \begin{bmatrix}
   3 & -5 & -4 \\
   -5 & -6 & -5 \\
   -4 & -5 & 3
   \end{bmatrix}
   \]

26. Diagonalise the matrix:
   \[
   \begin{bmatrix}
   5 & 4 \\
   1 & 2
   \end{bmatrix}
   \]

27. Check whether \(\{(-1, 1, 2), (2, -3, 1), (10, -14, 0)\}\) is a basis for \(\mathbb{R}^3\) over \(\mathbb{R}\) or not.

28. Let \(T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)\). Show that \(T\) is one-one. Does \(T\) map \(\mathbb{R}^2\) onto \(\mathbb{R}^3\)?
29. Let \( b_1 = [1 \ -3]^T, b_2 = [-2 \ 4]^T, c_1 = [-7 \ 9]^T, c_2 = [-5 \ 7]^T. \) Consider the bases of \( \mathbb{R}^2 \) given by \( B_1 = \{b_1, b_2\} \) and \( B_2 = \{c_1, c_2\} \). Find the change of co-ordinate matrix from \( B_2 \) to \( B_1 \) and the change of co-ordinate matrix from \( B_1 \) to \( B_2 \).

30. Let \( A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \) and \( B = \{b_1, b_2\}; \) for \( b_1 = [1 \ 1]^T, b_2 = [5 \ 4]^T \). Define \( T \) from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \) by \( T(x) = Ax \). Show that \( b_1 \) is an Eigen vector of \( A \). Is \( A \) diagonalizable?

31. Make a change of variable \( x = Py \), that transforms the quadratic form \( x_1^2 + 10x_1x_2 + x_2^2 \) in to a quadratic form without cross product term. Give \( P \) and the new quadratic form.

Section-IV

Answer any 2 questions from among the questions 32 to 35.
These questions carry 15 marks each.

32. a. Let \( T: \mathbb{R}^m \to \mathbb{R}^n \) be a linear transformation and let \( \{v_1, v_2, v_3\} \) be a linearly independent set in \( \mathbb{R}^n \). Show that the set \( \{T(v_1), T(v_2), T(v_3)\} \) is also linearly independent.

b. Find four bases for \( \mathbb{R}^3 \) over \( \mathbb{R} \), no two of which have a vector in common.

33. Define \( T \) from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \) by \( T(x) = Ax \) where \( A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \). Find a basis \( B \) for \( \mathbb{R}^2 \) with the property that \( [T]_B \) is diagonal.

34. Prove that \( \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix} \) is diagonalisable and find the diagonal form.

35. Prove that the vectors \( x_1 = (2 \ -1 \ 4), x_2 = (4 \ 0 \ 12) \) and \( x_3 = (0 \ 1 \ 2) \) are linearly dependent and find a relationship between them.

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