# UNIVERSITY OF KERALA Model Question Paper First Degree Programme in Physics Semester IV MM 1431.1 Mathematics – IV

# (Complex Analysis, Fourier Series, Fourier Transforms)

#### Time: 3 hours

Maximum Marks: 80

Section-I

#### All the first 10 questions are compulsory. They carry 1 mark each.

- 1. Find all roots of the equation  $\log z = i\pi/2$
- 2. Write down the principal value of log(-1)
- 3. State whether  $f(z) = \overline{z}$  is analytic or not.

4. Find the singular points of the function  $f(z) = \frac{2z+1}{z(z^2-1)}$ 

- 5. Write down the Taylor series for  $f(z) = \frac{1}{e^{-z}}$  at z = 0
- 6. Evaluate  $\int_C z^2 dz$  where C is any curve joining 0 to 1 + i
- 7. Find the residue of  $f(z) = \frac{z^2 1}{z^2 + z}$  at z = 0
- 8. Write down the Euler formulae for calculating the Fourier coefficients.
- 9. What is the standard form of Fourier series for an even function?
- 10. What are the sufficient conditions for the existence of Fourier transform.

## Section-II Answer any 8 questions from among the questions 11 to 22. These questions carry 2 marks each.

- 11. Prove that the real and imaginary parts of an analytic function are harmonic.
- 12. Show that an analytic function is constant if its modulus is constant.
- 13. Find an analytic function whose real part is  $u = e^{x}(x \cos y y \sin y)$ .

14. Evaluate  $\int_{C} (y - x - 3x^{2}i) dz$  where z = x + iy and C is the straight line joining 0 to 1 + i

- 15. State Cauchy's integral formula. Hence evaluate  $\int_C \frac{z^3 dz}{z^{-2}}$  where C is the circle |z| = 3
- 16. Expand  $f(z) = \frac{z}{(z-1)(z-3)}$  in Taylor series about z = 1
- 17. Expand  $f(z) = \frac{z-1}{z^3}$  in positive and negative powers of z 1
- 18. Find the nature of the singularity of the function  $f(z) = \sin\left(\frac{1}{z-1}\right)$

19. Find the residue of  $f(z) = \frac{1}{(z-1)^3}$  at its poles.

- 20. Find the Fourier series of f(x) = x;  $0 < x < 2\pi$
- 21. Find the Fourier series of f(x) = |x|; -2 < x < 2
- 22. Derive the Fourier transform of f'(x), the derivative of f(x)

# Section-III Answer any 6 questions from among the questions 23 to 31. These questions carry 4 marks each.

23. Show that the function  $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}; & z \neq 0\\ 0; & z = 0 \end{cases}$  is not differentiable at z = 0 even though

Cauchy-Riemann equations are satisfied there.

- 24. If a function is analytic, show that it is independent of  $\bar{z}$
- 25. Write different types of isolated singularities. Give one example each.
- 26. State Cauchy's residue theorem. Hence evaluate  $\int_C \frac{e^z dz}{\sin z}$  where C is the circle |z| = 1.
- 27. Evaluate  $\int_C \frac{(5z-2) dz}{z(z-1)}$  where *C* is the circle |z| = 2 described counter clockwise.
- 28. Evaluate  $\int_0^{2\pi} \frac{dz}{2+\cos\theta}$
- 29. Obtain the Fourier series of  $f(x) = x x^2$  in  $(-\pi, \pi)$ . Hence deduce  $\frac{1}{1^2} \frac{1}{2^2} + \frac{1}{3^2} \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$
- 30. Find the half range cosine series of  $f(x) = \begin{cases} \frac{2K}{L}x; & 0 < x < L/2\\ \frac{2K}{L}(L-x); & L/2 < x < L \end{cases}$
- 31. Find the Fourier transform of  $e^{-x^2/2}$ . What is your inference?

## Section-IV Answer any 2 questions from among the questions 32 to 35. These questions carry 15 marks each.

- 32. (i) Find an analytic function f(z) = u + iv whose real part is  $e^{x}(x \cos y y \sin y)$ .
- (ii) If  $u v = (x y)(x^2 + 4xy + y^2)$ , find an analytic function f(z) = u + iv in terms of z. 33. Expand  $f(z) = \frac{1}{(z-1)(z-2)}$  in a series of powers of z valid in:
  - (i) 0 < |z| < 1 (i) 1 < |z| < 2 (iii) |z| > 2

34. Find the Fourier series of  $f(x) = \frac{x^2}{2}$ ;  $-\pi < x < \pi$ . Hence deduce:  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ 

35. (i) Show that the Fourier transform is a linear operator

(ii) Find the Fourier transform of f(x), where

$$f(x) = \begin{cases} e^x; & x < 0\\ e^{-x}; & x > 0 \end{cases}$$