# UNIVERSITY OF KERALA
# Model Question Paper
## First Degree Programme in Physics and Computer Applications
## Semester IV
## MM 1431.4 Complementary Course for Physics and Computer Applications
## Mathematics – IV (Linear Transformations, Vector Integration and Complex Analysis)

**Time: 3 hours** **Maximum Marks: 80**

**Section-I**

**All the first 10 questions are compulsory. They carry 1 mark each.**

1. Define a contraction from $\mathbb{R}^2$ to $\mathbb{R}^2$
2. Let $A$ be a $7 \times 5$ matrix. What must $m$ and $n$ be in order to define $T: \mathbb{R}^m \to \mathbb{R}^n$ by $T(x) = Ax$
3. Write down the standard matrix corresponding to the transformation of horizontal shear.
4. Using Stoke's theorem find the value of $\int_C \vec{r}\,.d\vec{r}$ where $C$ is a simple closed curve in 2-space.
5. If $V$ is the volume enclosed by a surface $S$, then find the value of $\iint_S \vec{r}.\vec{n}\,dS$
6. Find all roots of the equation $\log z = \frac{i\pi}{2}$
7. Give an example of a function which is analytic everywhere.
8. If $u = x^2 - 2xy + ay^2$ is a harmonic function, find the value of $a$.
9. Find an analytic function whose real part is $u = x^2 - y^2$.
10. Evaluate $\int_C \frac{dz}{z-2}$ where $C$ is the circle $|z| = 3$.

**Section-II**

**Answer any 8 questions from among the questions 11 to 22.**
**These questions carry 2 marks each.**

11. Define a linear transformation and check whether the transformation $T$ is linear if $T$ is defined by: $T(x_1,\ x_2) = (2x_1 - 3x_2,\ x_1 + 4,\ 5x_2)$.
12. Let $T$ be the linear transformation defined by $T(e_1) = (1,4)$, $T(e_2) = (-2,9)$ and $T(e_3) = (3,-8)$, where $e_1, e_2$ and $e_3$ are columns of the $3 \times 3$ identity matrix. Check whether $T$ is one-one or not.
13. Find the dimension of the null space and the column space of:

$$\begin{pmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 0 & 0 & 1 & -3 & 7 & 0 \\ 0 & 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

14. Using Green's theorem evaluate $\int_C f(x)dx + g(y)dy$ where $C$ is an arbitrary simple closed curve in an open connected set $D$. What do you infer about the vector field $\vec{F}\,(x,y) = f(x)\,\hat{\imath} + g(y)\,\hat{\jmath}$.

15. Evaluate the flux of the vector field $\vec{F}(x, y, z) = z\,\hat{k}$ across the outward oriented sphere $x^2 + y^2 + z^2 = a^2$.
16. Find the work done by the force field $\vec{F}(x, y) = xy\,\hat{\imath} + x^2\hat{\jmath}$ on a particle that that moves along the parabola $x = y^2$ from $(0,0)$ to $(1,1)$
17. Prove that the real and imaginary parts of an analytic function are harmonic.
18. Find an analytic function $f(z) = u + iv$ whose real part is $e^x(x\cos y - y\sin y)$
19. Show that an analytic function is constant if its modulus is constant.
20. Evaluate $\int_C (y - x - 3x^2 i)\, dz$ where $z = x + iy$ and $C$ is the straight line joining $0$ to $1 + i$.
21. State Cauchy's integral formula. Hence evaluate $\int_C \frac{z^3 dz}{z-2}$ where $C$ is the circle $|z| = 3$.
22. Evaluate $\int_C \frac{(5z-2)\, dz}{z(z-1)}$ where $C$ is the circle $|z| = 2$ described counter clockwise.

**Section-III**
**Answer any 6 questions from among the questions 23 to 31.**
**These questions carry 4 marks each.**

23. Let $b_1 = [1 \quad -3]^{\mathrm{T}}$, $b_2 = [-2 \quad 4]^{\mathrm{T}}$, $c_1 = [-7 \quad 9]^{\mathrm{T}}$, $c_2 = [-5 \quad 7]^{\mathrm{T}}$. Consider the bases of $\mathbb{R}^2$ given by $B_1 = \{b_1, b_2\}$ and $B_2 = \{c_1, c_2\}$. Find the change of co-ordinate matrix from $B_2$ to $B_1$ and the change of co-ordinate matrix from $B_1$ to $B_2$.
24. Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ and $B = \{b_1, b_2\}$; for $b_1 = [1 \quad 1]^{\mathrm{T}}$, $b_2 = [5 \quad 4]^{\mathrm{T}}$. Define $T$ from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$. Show that $b_1$ is an Eigen vector of $A$. Is $A$ diagonalizable?
25. Evaluate the surface integral $\iint_\sigma xz\, dS$ where $\sigma$ is the part of the plane $x + y + z = 1$ that lies in the first octant. What happens if the integrand is $xy$?
26. Check whether $\vec{F}(x, y) = ye^{xy}\,\hat{\imath} + xe^{xy}\hat{\jmath}$ is conservative or not. If it is so, find the corresponding scalar potential.
27. Show that the function $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}; & z \neq 0 \\ 0; & z = 0 \end{cases}$ is not differentiable at $z = 0$ even though Cauchy-Riemann equations are satisfied there.
28. If a function is analytic, show that it is independent of $\bar{z}$.
29. Evaluate $\int_C \frac{(z^2+5)dz}{(z-2)^3}$ where $C$ is the circle $|z| = 3$ described counter clockwise.
30. Evaluate $\int_C \frac{(3z^2+2)dz}{(z-1)(z^2+9)}$ where $C$ is the circle $|z| = 4$ described counter clockwise.
31. Verify Cauchy's integral theorem for the function $f(z) = z^2$, the integral may be done along the circle $|z| = 1$

**Section-IV**

**Answer any 2 questions from among the questions 32 to 35.**

**These questions carry 15 marks each.**

32. Define $T$ from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$ where $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$. Find a basis $B$ for $\mathbb{R}^2$ with the property that $[T]_B$ is diagonal.

33. a. Let $T(x_1,\ x_2) = (3x_1 + x_2,\ 5x_1 + 7x_2,\ x_1 + 3x_2)$. Show that $T$ is one-one. Does $T$ map $\mathbb{R}^2$ onto $\mathbb{R}^3$?

    b. Check whether {(−1, 1, 2), (2, −3, 1), (10, −14, 0)} is a basis for $\mathbb{R}^3$ over $\mathbb{R}$ or not.

34. Consider the function $\vec{F}\,(x, y, z) = (x^2 - yz)\,\hat{\imath} + (y^2 - zx)\,\hat{\jmath} + (z^2 - xy)\hat{k}$ over the volume enclosed by the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b$ and $0 \le z \le c$. Verify Gauss's divergence theorem for $\vec{F}$.

35. a. Define an analytic function. Is $f(z) = \bar{z}$ analytic? Why?

    b. If $u - v = (x - y)(x^2 + 4xy + y^2)$, find an analytic function $f(z) = u + iv$ in terms of $z$

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