UNIVERSITY OF KERALA
Model Question Paper

First Degree Programme in Chemistry & Industrial Chemistry
Semester IV Complementary Course for Chemistry& Industrial Chemistry
MM 1431.2 Mathematics – IV (Linear Transformation, Vector Integration, Abstract Algebra, Fourier Series and Fourier Transforms)

Time: 3 hours **Maximum Marks: 80**

Section-I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Define a contraction from $\mathbb{R}^2$ to $\mathbb{R}^2$.
2. Let A be a 7×5 matrix. What must m and n be in order to define $T: \mathbb{R}^m \to \mathbb{R}^n$ by $T(x) = Ax$.
3. Write down the standard matrix corresponding to the transformation of horizontal shear.
4. Using Stoke's theorem find the value of $\int_C \vec{r}.d\vec{r}$ where C is a simple closed curve in 2-space.
5. If V is the volume enclosed by a surface S, then find the value of $\iint_S \vec{r}.\vec{n}\,dS$
6. Give an example of a finite group.
7. Give an example of a zero divisor in a ring.
8. Write down the Euler formulae for calculating the Fourier coefficients.
9. What is the standard form of Fourier series for an even function?
10. What are the sufficient conditions for the existence of Fourier transform.

Section-II

Answer any 8 questions from among the questions 11 to 22.
These questions carry 2 marks each.

11. Define a linear transformation and check whether the transformation T is linear if T is defined by: $T(x_1,\ x_2) = (2x_1 - 3x_2,\ x_1 + 4,\ 5x_2)$.
12. Let T be the linear transformation defined by $T(e_1) = (1,4)$, $T(e_2) = (-2,9)$ and $T(e_3) = (3,-8)$, where e_1, e_2 and e_3 are columns of the 3×3 identity matrix. Check whether T is one-one or not.
13. Find the dimension of the null space and the column space of:

$$\begin{pmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 0 & 0 & 1 & -3 & 7 & 0 \\ 0 & 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

14. Using Green's theorem evaluate $\int_C f(x)dx + g(y)dy$ where C is an arbitrary simple closed curve in an open connected set D. What do you infer about the vector field $\vec{F}(x,y) = f(x)\,\hat{\imath} + g(y)\,\hat{\jmath}$.

15. Evaluate the flux of the vector field $\vec{F}\,(x,y,z) = z\,\hat{k}$ across the outward oriented sphere $x^2 + y^2 + z^2 = a^2$.
16. Find the work done by the force field $\vec{F}\,(x,y) = xy\,\hat{\imath} + x^2\hat{\jmath}$ on a particle that that moves along the parabola $x = y^2$ from $(0,0)$ to $(1,1)$
17. Compute the subgroups $\langle 2\rangle$ and $\langle 5\rangle$ of the group $\langle \mathbb{Z}_6, +_6\rangle$
18. Define unit of a ring and give an example of it.
19. Is $(-1,2)$ and $(2,-4)$ a basis for $\mathbb{R}^2$ over $\mathbb{R}$?
20. Find the Fourier series of $f(x) = x;\ \ 0 < x < 2\pi$
21. Find the Fourier series of $f(x) = |x|;\ -2 < x < 2$
22. Derive the Fourier transform of $f'(x)$, the derivative of $f(x)$.

Section-III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. Let $b_1 = [1 \quad -3]^{\mathrm{T}},\ b_2 = [-2 \quad 4]^{\mathrm{T}}$, $c_1 = [-7 \quad 9]^{\mathrm{T}},\ c_2 = [-5 \quad 7]^{\mathrm{T}}$. Consider the bases of $\mathbb{R}^2$ given by $B_1 = \{b_1, b_2\}$ and $B_2 = \{c_1, c_2\}$. Find the change of co-ordinate matrix from B_2 to B_1 and the change of co-ordinate matrix from B_1 to B_2.
24. Let $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ and $B = \{b_1, b_2\}$; for $b_1 = [1 \quad 1]^{\mathrm{T}},\ b_2 = [5 \quad 4]^{\mathrm{T}}$. Define T from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$. Show that b_1 is an Eigen vector of A. Is A diagonalizable?
25. Evaluate the surface integral $\iint_\sigma xz\,dS$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant. What happens if the integrand is xy?
26. Check whether $\vec{F}\,(x,y) = ye^{xy}\,\hat{\imath} + xe^{xy}\hat{\jmath}$ is conservative or not. If it is so, find the corresponding scalar potential.
27. Define $*$ on the set of positive rational numbers Q^+ by $a * b = \frac{ab}{4}$. Show that $\langle Q^+, *\rangle$ is a group.
28. Check whether {(−1, 1, 2), (2, −3, 1), (10, −14, 0)} is a basis for $\mathbb{R}^3$ over $\mathbb{R}$ or not.
29. Obtain the Fourier series of $f(x) = x - x^2$ in $(-\pi, \pi)$.
30. Find the half range cosine series of $f(x) = \begin{cases} \frac{2K}{L}x; & 0 < x < L/2 \\ \frac{2K}{L}(L-x); & L/2 < x < L \end{cases}$
31. Find the Fourier transform of $e^{-x^2/2}$. What is your inference?

Section-IV
Answer any 2 questions from among the questions 32 to 35.
These questions carry 15 marks each.

32. a. Define T from $\mathbb{R}^2$ to $\mathbb{R}^2$ by $T(x) = Ax$ where $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$. Find a basis B for $\mathbb{R}^2$ with the property that $[T]_B$ is diagonal.

b. Show that $\mathbb{Z}_5$ is a field with respect to the operations, addition modulo 5 and multiplication modulo 5.

33. a. Find four bases for $\mathbb{R}^3$ over $\mathbb{R}$, no two of which have a vector in common.

b. Let $T(x_1,\ x_2) = (3x_1 + x_2,\ 5x_1 + 7x_2,\ x_1 + 3x_2)$. Show that T is one-one. Does T map $\mathbb{R}^2$ onto $\mathbb{R}^3$?

34. Consider the function $\vec{F}(x, y, z) = (x^2 - yz)\,\hat{\imath} + (y^2 - zx)\,\hat{\jmath} + (z^2 - xy)\hat{k}$ over the volume enclosed by the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b$ and $0 \le z \le c$. Verify Gauss's divergence theorem for $\vec{F}$.

35. a. Find the Fourier series of $f(x) = \frac{x^2}{2}; -\pi < x < \pi$. Hence deduce: $\sum_{n=1}^{\infty} \frac{1}{n^2}$

b. Find the Fourier transform of $f(x)$, where

$$f(x) = \begin{cases} e^{x}; & x < 0 \\ e^{-x}; & x > 0 \end{cases}$$
