

UNIVERSITY OF KERALA
Model Question Paper
First Degree Programme in Economics
Semester IV
MM 1431.5 Mathematics for Economics- IV

Time: 3 hours

Maximum Marks: 80

Section-I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Find the order of the differential equation $\frac{d^2y}{dx^2} + \sqrt{x + \left(\frac{dy}{dx}\right)^3} = 0$
2. Write the differential equation corresponding to $y = kx$
3. The integrating factor of $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x is
4. Find whether the differential equation $\frac{x}{x+y}$ homogeneous.
5. Find the differential equation corresponding to $y = Ae^x + Be^{-x}$
6. Find the general solution of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$
7. Find the particular integral of $y'' + y = e^x$
8. If $y = a^x$ satisfies the differential equation $\frac{dy}{dx} = y$, find a
9. The marginal cost of manufacturing an item is given by $\frac{dC}{dx} = 2 + 015x$. Find the total cost $C(x)$ given that $C(0) = 100$
10. What will be the order of the differential equation representing the system of parabolas $y = ax^2 + b$?

Section-II

Answer any 8 questions from among the questions 11 to 22.

These questions carry 2 marks each.

11. Show that $y = ae^{x^n}$ is an integral of $\frac{dy}{dx} = n y x^{n-1}$
12. Solve $(1 + x^2) \frac{dy}{dx} = 2xy$
13. Show that the equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is variables separable type and hence solve
14. Solve $\frac{dy}{dx} = xy^2 - x$
15. Find the differential equation of the family of curves $y = A \cos mx + B \sin mx$ where m is fixed and A, B are arbitrary constants.
16. Solve $y^2 dx = (xy - x^2) dy$
17. Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$

18. Solve $\frac{dy}{dx} + \frac{y}{x} = x^3$
19. Solve $(D^2 - 2D + 1)y = x^2$
20. Find the differential equation of all circles passing through the origin and having their centres on the x -axis.
21. If π is the total cost of an output x , it is known that the marginal cost is always equal to the average cost, show that π is a fixed multiple of x .
22. An individual's preference scale for two goods X and Y is defined by the marginal rate of substitution of Y for X , given by $R = \frac{X-a}{Y-b}$. Show that $U = (X - a)^2 + (Y - b)^2$ is a form of utility function.

Section-III

Answer any 6 questions from among the questions 23 to 31.

These questions carry 4 marks each.

23. If $y = \frac{a}{x-1}$, show that $(y - 1)dx + (x - 1)dy = 0$. Conversely, show that the differential equation has the integral shown. What are the curves represented?
24. Solve $\frac{dy}{dx} + y \tan x = \cos^3 x$
25. Solve $(2D^2 - D + 3)y = 0$
26. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^x + \sin x$
27. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = \sin 3x$
28. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2x + x^2$
29. The marginal cost is given by $MC = 25 + 30q - 9q^2$. The fixed cost is 55. Find the total cost, average cost and variable cost functions.
30. Find the demand function if the elasticity of demand is $\frac{a}{bx} - 1$, where a and b are positive constants.
31. Derive Domar's capital expansion model.

Section-IV

Answer any 2 questions from among the questions 32 to 35.

These questions carry 15 marks each.

32. (i) Find the differential equation corresponding to the family of curves, $y = (Ax + B)e^{-4x}$, where A and B are constants.
- (ii) Find the differential equation of which $y = e^x(A \cos 2x + B \sin 2x)$ is a solution, where A and B are constants.

33. Solve (i) $(x + y)dx + (x - y)dy = 0$

(ii) $xy^2 \frac{dy}{dx} = x^3 + y^3$

34. The change in price y with change in quantity x of a particular commodity is given by:

$\frac{dy}{dx} = \frac{-1-2x}{1+2y}$. Find the relationship between price and quantity demanded if the price is 4 when the quantity demanded is 2.

35. (a) The marginal revenue function for a product is given by $MR = \frac{6}{(x+2)^2} + 5$. Find the total revenue function. Also deduce the demand function.

(b) The price elasticity of demand curve $x = f(p)$ is of the form $a - bp$ where a and b are constants. Find the demand law.
