Model Question Paper

FOURTH SEMESTER B.TECH. EXAMINATIONS

Engineering mathematics -IV

(COMPLEX ANALYSIS AND LINEAR ALGRBRA)

(Common to AFRT)

(2013 Scheme)

Time: 3Hrs Max. Marks :100

Part A

Answer all questions. Each question carries 4 marks.

1. Define analytic function. State the necessary condition for a function \( f(z) \) to be analytic at a point. Use this to check whether \( f(z) = \bar{z} \) is not analytic.

2. Determine the region of the \( w \) plane in which the first quadrant of the \( z \) plane is mapped by the transformation \( w = z^2 \).

3. Evaluate \( \int \frac{1}{z-a} \, dz \), where \( C \) is a simple closed curve and the point \( z=a \) is (i) inside \( C \) (ii) outside \( C \).

4. Let \( H = \{(a - 3b, b - a, a) : a, b \in R\} \). Show that \( H \) is a subspace of \( R^3 \).

5. Find a unit vector orthogonal to \( (1,1,1) \) and \( (1,2,-3) \).

Part :B

Answer any one full question from each module. Each question carries 20 marks.

Module- I

6 a. If \( f(z) \) is an analytic function with constant modulus, then prove that \( f(z) \) is a constant.

b. If \( w = \phi + i \psi \) represents the complex potential of an electric field and \( \psi = x^2 - y^2 + \frac{x}{x^2 + y^2} \), determine the function \( \phi \).

c. Show that under the transformation \( w = \frac{z - i}{z + i} \), the real axis in the \( z \)-plane is mapped into the
circle $|w| = 1$.

7.a. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied at there.

b. Show that the function $u = e^{2z}(x \cos 2y - y \sin 2y)$ is harmonic. Find its harmonic conjugate and hence find the analytic function.

c. Find the bilinear transformation which maps the point $-1, i, 1$ of the $z$-plane on to the points $1, i, -1$ of the $w$-plane respectively

Module- II

8 a. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $0 < |z-1| < 1$.

b. Use Cauchy’s integral formula to evaluate $\int_C \frac{e^z}{(z+1)^2} \, dz$ where $C$ is $|z-1| = 3$.

c. By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{1}{5 - 4\cos \theta} \, d\theta$.

9 a. Discuss the singularities of (i) $\frac{1}{1-e^z}$, (ii) $ze^{-z}$.

b. Evaluate $\int_C \frac{e^{2z}}{z+1} \, dz$ where $C$ is $|z| = 2$.

c. Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)} \, dx$.

Module- III

10 a. Express $v=(2,7,-4)$ in $\mathbb{R}^3$ as a linear combination of the $u_1=(1,2,0)$, $u_2=(1,3,2)$ and $u_3=(0,1,3)$.

b. Find the dimension of the null space and column space of

\[
\begin{bmatrix}
-3 & 6 & -1 & 1 & -7 \\
-1 & -2 & 2 & 3 & -1 \\
2 & -4 & 5 & 8 & -4
\end{bmatrix}
\]
11a. Show that the mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$ is linear. Find a basis for the kernel of $T$.

b. Find a least square solution to the inconsistent system $AX = b$ where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

Module - IV

12 a. Find an orthonormal basis for the subspace spanned by $(1,2,1), (1,0,1)$ and $(3,1,0)$ in $\mathbb{R}^3$

b. Find a maxima or minima of $5x_1^2 + 5x_2^2 - 4x_1x_2$ subject to the constraint $x^Tx = 1$ where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

13a. Reduce the quadratic form $q = 8x_1^2 + 3x_2^2 + 3x_3^2 + 12x_1x_2 - 8x_2x_3 + 4x_1x_3$ to canonical form by orthogonal transformation. Examine the definiteness

b. Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$