Model Question for University Examination
University of Kerala

Sub: 13.506.4 Advanced Fluid Mechanics

Time allowed: Three hours Maximum: 100 Marks

PART A
(Answer all the questions; Each question carry 2 marks)

1. Write down the mathematical expressions for vorticity and angular velocity of a three-dimensional flow.

2. What is a material derivative? Write down an equation for material derivative.

3. Write Reynolds transport equation by clearly mentioning meanings for each variable involved.


5. What is a potential flow? Write the equation governing a potential flow.

6. The complex flow potential for a flow is given as \( f(z) = U_\infty e^{-ia} \). Write down its velocity and identify the type of flow.

7. Write down the expanded form of N-S equation in polar coordinates.

8. Write down the momentum integral equation for flow over a flat plate.

9. Define Reynolds number.

10. Explain laminar flow separation.
PART B
(Answer any one question from each module; Each question carry 20 marks.)

Module-I

11. (a) The motion of a fluid is described by the following Lagrangian coordinates

\[ x = x_0 \left( 1 + \frac{t}{\tau} \right); \quad y = y_0 \left( 1 + 2 \frac{t}{\tau} \right); \quad z = z_0 \left( 1 + \frac{t^2}{\tau^2} \right) \]

where \( \tau \) is a constant. Find (i) the velocity field (ii) Find the position of the particle at 
\( t = 3\tau \), having location \((a, b, c)\) at \( t = \tau \). (10 marks)

(b) Consider the motion of fluid with velocity and density \( \vec{V} = k(x\hat{i} - y\hat{j}) \) and \( \rho = \rho_0 + Ay e^{kt} \), respectively. Where \( k, \rho_0 \) and \( A \) are constants. Find out the rate of change of density of individual fluid particles? (10 marks)

12. (a) Find the Lagrangian coordinate functions for \( x, y \) and \( z \) corresponding to the Eulerian velocity field \( u = -Ax, v = By \) and \( w = 0 \). Given that \( A \) and \( B \) are positive constants. (10 marks)

(b) Find the equation of stream lines for the flow given by \( u = u_0, v = v_0 \sin \Omega t, w = w_0 \), where \( u_0, v_0, w_0 \) and \( \Omega \) are constants. (10 marks)

Module-II

13. (a) Derive an equation for complex flow potential of a doublet flow. (10 marks)

(b) Write down the equation of complex flow potential for flow over a circular cylinder. Hence derive the equation for radial and tangential components of velocity at any point in the field. (10 marks)

14. Using conformal transformations derive the complex flow potential for flow against a vertical flat plate. (20 marks)

Module-III

15. Simplify the Navier-Stokes equation in polar cylindrical coordinates for the case of developed flow though a pipe by clearly mentioning the assumptions. Solve this equation and find out (i) the velocity distribution and (ii) wall shear stress (10+5+5 marks)

16. Derive an equation for friction factor in laminar flow through a pipe. (20 marks)
17. (a) Consider two-dimensional laminar boundary layer flow over a flat plate aligned with the direction of a uniform oncoming free stream. The velocity profile in the boundary layer is approximated by the sinusoidal distribution $u/U_\infty = \sin(\pi y/2\delta)$ for $y$ lying between 0 and $\delta$ and $u/U_\infty = 1$ for $y > \delta$. Determine the variation of boundary layer thickness and the total skin-friction drag on a plate of length $L$ and width $W$. (15 marks)

(b) Discuss any four characteristic features of turbulent flow. (5 marks)

18. (a) Derive general momentum integral equation involving pressure gradient. (10 marks)

(b) Derive an ordinary differential equation in terms of a similarity variable for boundary flow over a flat plate. (10 marks)