UNIVERSITY OF KERALA
Model Question Paper
First Degree Programme
Semester VI Core Course
MM: 1642 Linear Algebra

Time: 3 Hours **Maximum Marks: 80**

Section I

All the first 10 questions are compulsory. Each carries 1 mark.

1. Write an equation of the line through the vector $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ parallel to the vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$.
2. Find an angle between the vectors $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$. and $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.
3. Find the projection of the vector $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ to the line along $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
4. Let A and B be two linear transformations. If B is a reflection in the $X-$ axis and A is a reflection in the $Y-$ axis. Then what is AB?
5. True / False : If vector A is a scalar multiple of vector B, then B is a scalar multiple of A.
6. True / False : The square of a symmetric matrix is symmetric.
7. Define an eigenvalue of a linear transformation.
8. Define an isometry transformation.
9. What kind of locus is defined by the equation $x^2 + y^2 - z^2 = 0$.
10. Define a regular simplex.

Section II

Answer any 8 questions from this section.
Each question carries 2 marks

11. Let T be the transformation with matrix $\begin{pmatrix} 2 & 1 \\ 0 & 5 \end{pmatrix}$. Find the matrix T^{-1}.
12. Let E be a linear transformation such that $E^2 = E$. What are the eigenvalues of E?
13. Prove : If T_1 and T_2 are two length preserving transformations then so is T_1T_2.
14. Prove : If $\overline{A}$ and $\overline{B}$ are linearly independent vectors, then $\overline{A} \times \overline{B} = \overline{0}$
15. In terms of co-ordinates of $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, calculate the images of R^1_π and $R^1_{\frac{\pi}{4}}$

16. Let T be a transformation of $\mathbb{R}^3$, such that $T(X) = 0$ implies $X = 0$. Prove that T has an inverse.

17. Classify the curve $2xy - y^2 = 1$.

18. Prove that $(e_{3\ 2}^{s})^{-1} = e_{3\ 2}^{-x}$.

19. Find the characteristic equation for the transformation with matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{pmatrix}$.

20. Prove : the only eigenvalues of an isometry of $\mathbb{R}^3$ are 1 or -1.

21. Let a 3×3 matrix m be the matrix of an isometry, show that m is an orthogonal matrix.

22. Show that the vectors $\begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix}, \begin{pmatrix} u \\ -v \\ -x \\ y` \end{pmatrix}, \begin{pmatrix} v \\ u \\ -y \\ -x \end{pmatrix}$ are mutually orthogonal.

Section III

Answer any 6 questions from this section.

Each question carries 4 marks.

23. Let A be a linear transformation and L be a straight line. Then prove that the image of L under A is either a straight line or a single point.

24. **(a)** Prove : If T_1 and T_2 are two reflections, then T_1T_2 is reflection.

    (b) Prove : If T_1 is a rotation and T_2 is a reflection, then T_1T_2 is reflection.

25. Show that the transformation T with matrix $\begin{pmatrix} a & b & c \\ 2a & 2b & 2c \\ 3a & 3b & 3c \end{pmatrix}$ has no inverse for any values of a, b, c.

26. Let S, T be two linear transformations. Prove that if the product ST has an inverse, then both S and T has inverse.

27. Find all solutions of the non homogeneous system $x_1 + x_2 + x_3 = 1,\ 2x_1 - x_2 = 5$ and $5x_1 + 2x_2 + 3x_3 = 8$.

28. Prove that if T is an isometry of $\mathbb{R}^3$, then $\det(T) = \pm 1$.

29. Classify the quadratic surface $x^2 + 10xz + y^2 + 6yz + z^2 = 1$.

30. Find a nonzero vector $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ in $\mathbb{R}^3$ which is orthogonal to each of the vectors $\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

31. Find the coefficients a, b, c of a parabola $y = ax^2 + bx + c$ which passes through the points $(1, 6),\ (2, 4),\ (3, 0)$.

Section IV

Answer any 2 questions from this section.

Each question carries 15 marks.

32. Prove that a real number t is an eigenvalue of the linear transformation A if and only if t is a root of the characteristic equation of A.

Find all eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$.

33. **(a)** Let A be a linear transformation with linearly independent eigenvectors $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$, corresponding to the eigenvalues t_1 and t_2. Then $A = PDP^{-1}$ where $m(P) = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}$ and $m(D) = \begin{pmatrix} t_1 & 0 \\ 0 & t_2 \end{pmatrix}$.

(b) Calculate $\begin{pmatrix} 3 & 0 \\ 4 & 2 \end{pmatrix}^6$.

34. **(a)** Prove that a linear transformation A on $\mathbb{R}^3$ preserves orientation if and only if $\det(m(A)) > 0$.

(b) Let T be an orientation-preserving linear transformation of $\mathbb{R}^3$. Prove that if $\prod$ is any parallelepiped, then $\text{volume}(T(\prod)) = (\det(m(T)))(\text{volume}(\prod))$.

35. State and prove spectral theorem in $\mathbb{R}^3$.