KERALA UNIVERSITY

Model Question Paper- M. Sc. Examination

2021 admission onwards

Branch : Mathematics

MM 234- GRAPH THEORY

Time: 3 hours | Max. Marks:75

Part A

Answer any 5 questions from among the questions 1 to 8

Each question carries 3 marks

1. Prove that every non-trivial connected graph contains at least two vertices that are not cut-vertices.
2. Find $\lambda(K_n)$
3. Give an example of a graph G such that both G and $\bar{G}$ are Eulerian.
4. Find the cyclic 1-factorization of K_6.
5. Explain the problem of Five princes and the problem of Five Palaces.
6. Find $r(K_3, K_3)$
7. Define peripheral vertex, eccentric vertex and boundary vertex. Give example for each.
8. Prove that a connected graph G of order $n \geq 2$ has locating number $n-1$ if and only if $G \equiv K_n$

$5 \times 3 = 15$

Part B

Answer all questions from 9 to 13

Each question carries 12 marks

9. A. i. Prove that isomorphism is an equivalence relation on the set of all graphs.
   ii. Determine $Aut(C_5)$

OR

   B. i. Prove that a graph of order at least 3 is nonseperable if and only if every two vertices lie on a common cycle.
   ii. For every graph G, prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$

10. A. i. Prove that a connected graph G contains an Eulerian trail if and only if exactly two vertices of G have odd degree and each Eulerian trail of G begins at one of these odd vertices and ends at the other.
    ii. Prove that Petersen graph is non-Hamiltonian.

OR

   B. i. State and prove Ore's theorem.

ii. Define $h(G)$ and $h^*(G)$ and prove that for every connected graph G, $h(G) = h^*(G)$

11. A. i. Prove that a nontrivial connected graph G has a strong orientation if and only if G contains no bridge.

ii. Prove that every tournament contains a Hamiltonian path

OR

B. i. Define edge independence number $\beta_1(G)$ and edge covering number $\alpha_1(G)$. Prove that for every graph G of order n containing no isolated vertices, $\alpha_1(G) + \beta_1(G) = n$

ii. State and prove Petersen's theorem.

12. A. i. Prove that for every graph G, $\chi(G) \leq 1 + \max\{\delta(H)\}$, where the maximum is taken over all induced subgraphs H of G.

ii. State Vizing's theorem. If G is a graph or odd order n and size m with $m > \dfrac{(n-1)\Delta(G)}{2}$, then prove that $\chi_1(G) = 1 + \Delta(G)$

OR

B. i. For every integer $k \geq 3$, prove that there exists a triangle-free graph with chromatic number k.

ii. Prove that every graph of order $n \geq 3$ and size at least $\binom{n-1}{2} + 2$ is Hamiltonian

13. A. i. Define center of a graph and prove that center of every connected graph G is a subgraph of some block of G.

ii. Prove that a nontrivial graph G is the eccentric subgraph of some graph if and only if every vertex of G has eccentricity 1 or no vertex of G has eccentricity 1.

OR

B. i. For a connected graph G, prove that a vertex v is a boundary vertex of G if and only if v is not an interior vertex of G.

ii. Define detour distance and prove that detour distance is a metric on the vertex set of every connected graph.

. $5 \times 12 = 60$