UNIVERSITY OF KERALA

Model Question Paper- M. Sc. Examination 2021 admission onwards

Branch : Mathematics

Course Code-MM 243 Course Name-Difference Equations

Time: 3 hours Max. Marks:75

Part A

Answer any 5 questions from among the questions 1 to 8

Each question carries 3 marks

1. Prove that Δ^{-1} is linear.
2. Consider the difference equation $y(n+3) - \frac{n}{n+1}y(n+2) + ny(n+1) - 3y(n) = n$, where $y(1) = 0, y(2) = -1$ and $y(3) = 1$. Find $y(6)$.
3. For each $x_0 \in \mathbb{R}^k$ and $n_0 \in \mathbb{Z}+$ there exists a unique solution $x(n, n_0, x_0)$ of $x(n+1) = A(n)x(n)$ with $x(n_0, n_0, x_0) = x_0$.
4. State and prove Abel's formula.
5. Find the Z-transform of the sequence $\{\sin(\omega n)\}$.
6. Write a short note on self-adjoint second order difference equations.
7. Show that $\left(\frac{n}{t^2+n^2}\right)^n = O\left(\frac{1}{t^n}\right), n \to \infty$, for $n \in \mathbb{Z}^+$.
8. Define ordinary dichotomy. . $5 \times 3 = 15$

Part B

Answer all questions from 9 to 13

Each question carries 12 marks

9. .

A. . State and prove Abel's lemma.

OR

B. Solve the difference equations

a). $(E-3)(E+2)y(n) = 5(3^n)$.

b). $y(n+2) + 4y(n) = 8(2^n)\cos\left(\frac{n\pi}{2}\right)$

10. .

A. . Find the general solution of $x(n+1) = Ax(n)$, where $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.

OR

B. For every fundamental matrix $\Phi(n)$ of system $x(n+1) = A(n)x(n)$, there exists a nonsingular periodic matrix $P(n)$ of period N such that $\Phi(n) = P(n)B^n$.

11. .

A. . Write a note on the properties of Z-transform.

OR

B. Solve the difference equation $x(n+4)+9x(n+3)+30x(n+2)+44x(n+1)+24x(n) = 0$, $x(0) = 0, x(1) = 0, x(2) = 1, x(3) = 10$.

12. .

A. . State and prove Sturm separation theorem.

OR

B. If $b(n)b(n+1) \leq (4-\epsilon)p^2(n)$ for some $\epsilon > 0$ and for all $n \geq N$, then every solution of $p(n)x(n+1) + p(n-1)x(n-1) = b(n)x(n)$ is oscillatory.

13. .

A. . Suppose that system $x(n+1) = D(n)x(n)$ possesses an ordinary dichotomy and the following condition holds: $\sum_{n=n_0}^{\infty} ||B(n)|| < \infty$. Then for each bounded solution $x(n)$ of $x(n+1) = D(n)x(n)$ there corresponds a bounded solution $y(n)$ of $y(n+1) = (D(n)+B(n))y(n)$ given by $y(n) = x(n) + \sum_{j=n_0}^{n-1} \Phi_1(n)\Phi^{-1}(j+1)B(j)y(j) - \sum_{j=n}^{\infty} \Phi_2(n)\Phi^{-1}(j+1)B(j)y(j)$. The converse also holds; for each bounded solution $y(n)$ of $y(n+1) = (D(n)+B(n))y(n)$ there corresponds a bounded solution $x(n)$ of $x(n+1) = D(n)x(n)$.

OR

B. Suppose that $\lim_{n\to\infty} \frac{x(n+1)}{x(n)} = \lambda$.

(a) If $\lambda \neq 0$, then $x(n) = \pm\lambda^n e^{n\nu(n)}$ for some null sequence $\nu(n)$.

(b) If $\lambda = 0$, then $|x(n)| = e^{-n/\mu(n)}$ for some positive null sequence $\mu(n)$.

. $5 \times 12 = 60$