

Reg. No.: .....



Name: .....

**University of Kerala**

U8763

Second Semester FYUGP Degree Examination, April 2025

Discipline Specific Core Course

**MATHEMATICS**

**UK2DSCMAT106 - Linear Algebra and Graph Theory**

Academic Level: 100-199

**Time: 2 Hours(120 Mins)**

**Max. Marks: 56**

**Part A.6 Marks:Time 5 Minutes.(Cognitive Level :Remember(RE)/Understand(UN)) Objective Type.1 mark each, Answer all questions**

| Qn No. | Question                                                                                        | CL | CO |
|--------|-------------------------------------------------------------------------------------------------|----|----|
| 1      | Define Eigen value of a Matrix.                                                                 | RE | 2  |
| 2      | If 2 and 3 are eigen values of a matrix $A$ , then what are the eigen values of $A^T$ .         | RE | 2  |
| 3      | Write the transpose of the matrix $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 5 \end{bmatrix}$ . | UN | 1  |
| 4      | Give an example of a tree having 3 verices.                                                     | UN | 2  |
| 5      | The transpose of a transpose of a matrix is ....                                                | UN | 3  |
| 6      | Draw two isomorphic graphs.                                                                     | UN | 1  |

**Part B.10 Marks.Time:20 Minutes (Cognitive Level:Understand(UN)/Apply(AP))Two-three sentences.2 marks each.Answer all questions**

| Qn No. | Question                                                                                                               | CL | CO |
|--------|------------------------------------------------------------------------------------------------------------------------|----|----|
| 7      | Write two properties of an incidence matrix.                                                                           | UN | 3  |
| 8      | Give an example for a skew symmetric matrix.                                                                           | UN | 2  |
| 9      | Compute the characteristic equation of $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ .                            | AP | 2  |
| 10     | If in a graph $G$ there is one and only one path between every pair of vertices, then prove that $G$ is a tree.        | AP | 4  |
| 11     | Compute the Eigen values of $A$ and $A^T$ , if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ | AP | 2  |

**Part C.16 Marks.Time:35 Minutes.(Cognitive Level :Apply(AP)/Analyse(AN))Short Answer.4 marks each, Answer all 4 questions,choosing among options \* within each question**

| Qn No. | Question                                                                                                                                                                                                                                                                                                                                                                                                   | CL | CO   |
|--------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|------|
| 12     | <p>A)</p> <p>Check whether <math>\begin{bmatrix} \cos \theta &amp; \sin \theta \\ -\sin \theta &amp; \cos \theta \end{bmatrix}</math> is orthogonal.</p> <p>OR</p> <p>B)</p> <p>Explain the algorithm for the shortest spanning tree.</p>                                                                                                                                                                  | AP | 2, 3 |
| 13     | <p>A)</p> <p>Prove that the number of vertices of odd degree in a graph is always even.</p> <p>OR</p> <p>B)</p> <p>Consider the matrices <math>A = \begin{bmatrix} -2 &amp; 2 \\ 2 &amp; 1 \end{bmatrix}</math> and <math>B = \begin{bmatrix} 2 &amp; 1 \\ 3 &amp; 4 \end{bmatrix}</math>. Prove the following</p> <p>(i) <math>(A + B)^T = AT + B^T</math></p> <p>(ii) <math>(AB)^T = B^T A^T</math>.</p> | AP | 3, 2 |
| 14     | <p>A)</p> <p>Prove that a graph <math>G</math> with <math>n</math> vertices <math>n - 1</math> edges and no circuits is connected.</p> <p>OR</p> <p>B)</p> <p>Find the eigen values of <math>A = \begin{bmatrix} 4 &amp; 2 \\ 1 &amp; -2 \end{bmatrix}</math></p>                                                                                                                                          | AN | 3, 2 |
| 15     | <p>A)</p> <p>Show that there is one and only one path between every pair of vertices in a tree <math>T</math>.</p> <p>OR</p> <p>B)</p> <p>Draw a spanning tree of <math>K_4</math>.</p>                                                                                                                                                                                                                    | AN | 3, 3 |

**Part D.24 Marks.Time: 60 Minutes.(Cognitive Level :Analyse(AN)/Evaluate(EV)/Create(CR)) Long Answer 6 Marks each.Answer all 4 questions choosing among options \* within each question**

| Qn No. | Question                                                                                      | CL | CO   |
|--------|-----------------------------------------------------------------------------------------------|----|------|
| 16     | <p>A)</p> <p>Prove that a tree with <math>n</math> vertices has <math>n - 1</math> edges.</p> | AN | 3, 2 |

| Qn No. | Question                                                                                                                                                                                                                                                                                                                                                  | CL | CO   |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|------|
|        | <p>OR</p> <p>B)</p> <p>Calculate the Eigen value and Eigen vector of the matrix <math>\begin{bmatrix} 2 &amp; 1 \\ 1 &amp; 2 \end{bmatrix}</math></p>                                                                                                                                                                                                     |    |      |
| 17     | <p>A)</p> <p>Explain incidence matrix and adjacency matrix of a graph with an example. Write a property of each of these matrices.</p> <p>OR</p> <p>B)</p> <p>Express the matrix <math>A = \begin{bmatrix} 1 &amp; 2 &amp; -3 \\ 4 &amp; 2 &amp; -1 \\ 0 &amp; 4 &amp; -2 \end{bmatrix}</math> as the sum of a symmetric and a skew symmetric matrix.</p> | EV | 4, 2 |
| 18     | <p>A)</p> <p>Solve</p> $\begin{aligned} x + 2y + z &= 3 \\ 2x + 3y + 2z &= 5 \\ 3x - 5y + 5z &= 2 \end{aligned}$ <p>OR</p> <p>B)</p> <p>Find all solutions or indicate that no solution exists</p> $\begin{aligned} 4y + z &= 0 \\ 12x - 5y - 3z &= 34 \\ -6x + 4z &= 8 \end{aligned}$                                                                    | EV | 2, 2 |
| 19     | <p>A)</p> <p>(a) Define a Euler Graph. Give an example.</p> <p>(b) Prove that a connected graph <math>G</math> is an Euler graph if and only if all vertices of <math>G</math> are of even degree.</p> <p>OR</p> <p>B)</p> <p>Show that every tree has either one or two centers. Give an example to illustrate this.</p>                                 | CR | 3, 3 |