

Reg. No.:



Name:

University of Kerala

U8758

Second Semester FYUGP Degree Examination, April 2025

Discipline Specific Core Course

MATHEMATICS

UK2DSCMAT101 - Integration and Multivariate Calculus

Academic Level: 100-199

Time: 1 Hour 30 Minutes(90 Mins.)

Max. Marks: 42

Part A. 6 Marks.Time:6 Minutes.(Cognitive Level:Remember(RE)/Understand(UN)) Objective Type. 1 Mark
Each.Answer all questions

Qn No.	Question	CL	CO
1	Find $r'(t)$ if $r(t) = (4 + 5t)\mathbf{i} + (t - t^2)\mathbf{j}$.	RE	1
2	$\int \sec x \tan x dx = \dots\dots\dots$	RE	1
3	Suppose that the function $f(x, y)$ is continuous and has second order partial derivatives and $(0, 0)$ is a critical point for $f(x, y)$. State whether $f(x, y)$ has maxima or minima at $(0, 0)$ if $f_{xx}(0, 0) = -2$, $f_{xy}(0, 0) = 2$, $f_{yy}(0, 0) = 2$.	UN	1
4	Find $\int_0^1 (e^{2t} \mathbf{i} + e^{-t} \mathbf{j} + t \mathbf{k}) dt$.	UN	1
5	Find $\int \frac{\sec x + \tan x}{\cos x} dx$.	UN	1
6	Let $f(x, y) = \frac{x-y}{x+y+1}$. Then $f(y+1, y) = \dots\dots\dots$	UN	1

Part B.8 Marks.Time:24 Minutes.(Cognitive Level:Understand(UN)/Apply(AP))Short Answer. 2 marks each.Answer all questions

Qn No.	Question	CL	CO
7	Find the parametric equation of the tangent line to the circular helix $x = \cos t, y = \sin t, z = t$ at the point $t = \pi$	UN	2
8	Suppose that $z = 5x^3y^2, x = t^3, y = t^5$. Use the chain rule to find $\frac{dz}{dt}$	UN	3

Qn No.	Question	CL	CO
9	Evaluate $\int_0^{\frac{\pi}{8}} (\sin^5 2x \cos 2x) dx$.	AP	2
10	Find parametric equation of the tangent lines to the circular helix $x = \cos t$, $y = \sin t$, $z = t$ where $t = t_0$, and use that result to find the parametric equations for the tangent lines at the point where $t = \pi$.	AP	2

Part C. 28 Marks. Time: 60 Minutes (Cognitive Level: Apply (AP)/Analyse (AN)/Evaluate (EV)/Create (CR)) Long Answer. 7 marks each. Answer all 4 Questions choosing among options * within each question

Qn No.	Question	CL	CO
11	A) Calculate $\int x^2 \sqrt{x-1} dx$. OR B) (a) Sketch the region enclosed by the curves and find its area : $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$, $x = 1$. (b) Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x-axis.	AP	2, 3
12	A) Find relative extrema and saddle points, if any, of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$. OR B) a) Let $\vec{r}_1(t) = \tan^{-1} t \ i + \sin t \ j + t^2 \ k$ and $\vec{r}_2(t) = (t^2 - t) \ i + (2t - 2) \ j + \ln t \ k$. The graphs of $\vec{r}_1(t)$ and $\vec{r}_2(t)$ intersect at the origin. Find the degree measure of acute angle between the tangent lines to the graphs of $\vec{r}_1(t)$ and $\vec{r}_2(t)$ at the origin. b) Let $\vec{r}(t)$ be a differential vector valued function with norm $\ \vec{r}(t)\$ is constant for all t. Prove $\vec{r}(t)$ and its derivative $\vec{r}'(t)$ are orthogonal vectors.	AN	3, 3
13	A) (a) Evaluate $\int (1 + \sin t)^9 \cos t dt$ (b) Evaluate $\int_2^5 (2x - 5)(x - 3)^9 dx$. OR B)	EV	2, 3

Qn No.	Question	CL	CO
	Find $\mathbf{T}(t)$ and $\mathbf{N}(t)$ for the vector $\mathbf{r}(t) = (t^2 + 1)\mathbf{i} + t\mathbf{j}$ at $t = 2$.		
14	A) Let $w = 4x^2 + 4y^2 + z^2$, $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$. . Use appropriate form of chain rule to find $\frac{\partial w}{\partial \rho}$, $\frac{\partial w}{\partial \theta}$ and $\frac{\partial w}{\partial \phi}$. OR B) Determine the dimensions of a rectangular box, open at the top, having a volume of $32ft^3$, and requiring the least amount of material for its construction	CR	4, 3