

**SEMESTER : 2**

**COURSE CODE : ST 221**

**COURSE TITLE : PROBABILITY THEORY II**

**COURSE OUTCOMES**

**On completion of the course, students should be able to:**

CO1 : Calculate the expectation and moments of random variables.

CO2 : Identify the applications of various moment inequalities.

CO3 : Explain the concept of convergence and check for the of convergence of a given sequences of random variables.

CO4 : Find the expressions for the characteristic function of a random variable and verify its properties.

CO5 : Apply the various laws of large numbers to sequences of random variables.

**MODULE OUTCOMES**

<b>Sl. No:</b>	<b>Outcomes</b>	<b>Taxonomy Level</b>
Module I	On completion of each module, students should be able to: MO1 : Calculate the mathematical expectation, moments and generating functions of random variables.	Evaluate
	MO2 : Articulate the various moment inequalities	Understand
	MO3 : Apply the various moment inequalities to various distributions.	Apply
Module II	MO1 : Articulate and appraise stochastic convergence of sequence of random variables.	Understand
	MO2 : Apply the concepts of convergence to sequences of random variables.	Apply
	MO3 : Construct counter examples for not satisfying certain convergence implications.	Create

Module III	MO1 : Derive expressions for the characteristic function for various distributions.	Evaluate
	MO2 : Find moments using characteristic function.	Evaluate
	MO3 : Derive expressions for the probability density function corresponding to a given characteristic function'	Evaluate
	MO4 : Articulate the various theorems associated with the characteristic function and identify their applications.	Apply
Module IV	MO1 : State and prove the various laws of large numbers.	Understand
	MO2 : Apply the laws to sequences of random variables.	Apply
	MO3 : State and prove various central limit theorems.	Understand
	MO4 : Applications of various central limit theorem.	Apply

### Module 1

Expectation of random variables and its properties, moments and factorial moments, probability generating function, moment generating function and cumulant generating function. Inequalities- Markov, Chebychev's, Lyapunov (for moments), Jensen, Holder's,  $C_r$ - inequality, Minkowski and basic inequality.

### Module II

Stochastic convergence of sequence of random variables: - convergence in probability, almost sure convergence, convergence in  $p^{\text{th}}$  mean, weak and complete convergence of distribution functions and their interrelations. Slutsky's theorem and its applications. Helly-Bray lemma (statement only) and Helly-Bray theorem (statement only).

### Module III

Characteristic function (c.f.) and their elementary properties, uniform continuity and non-negative definiteness of characteristic function. Uniqueness theorem, Inversion theorem (without proof), Fourier inversion theorem, Convolution theorem, Levy's continuity theorem (without proof) and Bochners theorem (without proof).

## **Module IV**

Stochastic series of sequence of random variables: - Law of large numbers, weak law of large numbers due to Bernoulli, Tchebychev and Khintchine. Kolmogorov inequality (statement only), Kolmogorov three-series theorem(statement only). Strong law of large numbers- Kolmogorov's strong law of large numbers for independent random variables (statement and applications only). Kolmogorov's strong law of large numbers for independent and identically distributed random variables (statement and applications only). Central limit theorem: Classical Central limit theorem, De Moivre-Laplace Central limit theorem, Statement of Lyapunov Central limit theorem and Lindberg-Feller Central limit theorem. Applications of various central limit theorems.

## **Text books**

1. Bhat, B.R. (1991).Modern Probability Theory, 2<sup>nd</sup>Edn., Wiley Eastern Ltd., New Delhi.
2. Laha, R.G and Rohatgi, V.K. (1979). Probability Theory, John Wiley, New York.
3. Rohatgi, V.K. and Saleh, Ehsanes (2014). An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern Ltd.

## **References**

1. Cacoullos, T. (1989). Exercise in Probability, Springer-Verlag, New-York.
2. Feller W. (1968). Introduction to Probability Theory and Its Applications Vol. 1 and 2, John Wiley, New York.
3. Gnedenko, B.V. (1969). The Theory of Probability, Mir Publishers, Mosko.
4. Loeve, M. (1968). Probability Theory Allied East-West Press.
5. Mukhopadhyay, P. (2011). An Introduction to the Theory of Probability, World Scientific Publishing Company.

6. Roussas, G.G.(2014). An Introduction to Measure- Theoretic Probability, Academic Press, USA.

**COURSE CODE : ST 222**

**COURSE TITLE : DISTRIBUTION THEORY**

### **COURSE OUTCOMES**

On completion of the course, students should be able to:

CO1. Understand various specifications such as probability density functions and cumulative distribution functions etc, of the probability distribution of a random variables.

CO2. Derive various generating functions of random variables such as probability generating function, moment generating functions, characteristic functions, etc.

CO3. Find out characteristics of random variables like moments from either probability density (mass) functions or the generating functions.

CO4. Understand the probability distribution of bivariate random variables and the terms marginal distributions, conditional distributions, marginal, joint and conditional moments.

CO5. Apply transformation of variable technique for finding the distribution of functions of random variables and solve related problems

CO6. Understand the properties and applications of some standard univariate and bivariate probability distributions for both discrete and continuous random variables.

CO7. Explain the concept of order statistics and solving problems related to it.

CO8. Explain different sampling distributions, and their properties and applications

### MODULE OUTCOMES

Sl. No:	Outcomes	Taxonomy level
Module I	<p>On completion of each module, students should be able to:</p> <p>M01. Explain the basic concepts of pgf, mgf and characteristic functions</p> <p>M02 Evaluate characteristics of random variables</p> <p>M03. Find out the joint, marginal , conditional distributions and characteristics of a bivariate distribution</p> <p>M04. Apply transformation of variable technique for finding the distribution of functions of random variables and solve related problems</p>	<p>Understand</p> <p>Apply</p> <p>Apply</p> <p>Analysis</p>
Module II	<p>M01. Understand the properties of standard univariate discrete distributions.</p> <p>M02 Articulate the connections between discrete distributions</p> <p>M03. Explain the how to apply the standard discrete distributions in various situations</p> <p>M04. Articulate multivariate discrete probability distributions such as multinomial and bivariate Poisson distribution</p>	<p>Understand</p> <p>Analysis</p> <p>Apply</p> <p>Analysis</p>
Module III	<p>M01. Understand the properties standard univariate continuous distributions.</p> <p>M02 Articulate the connections between continuous distributions</p> <p>M03. Explain the how to apply the standard continuous distributions in various situations</p> <p>M04. Understand the bivariate normal distributions and its applications</p>	<p>Understand</p> <p>Analysis</p> <p>Apply</p> <p>Understand</p>

Module IV	M01. Explain the concept of order statistics	Understand
	M02. Find the joint distribution of two order statistics	Apply
	M03. Calculate Distribution of functions of two order statistics	Apply Understand
	M04. Explain different sampling distributions	Apply
	M05. Find out expressions for probability density function of sampling distribution	Apply
	M06 Understand the applications of sampling distributions	

## COURSE CONTENT

### Module I

Generating functions (concepts and examples only) - probability generating function, moment generating function, cumulant generating function and characteristic function; bivariate distributions- joint, marginal and conditional distribution. Independence of random variables, bivariate mgf, pgf and moments, functions of random variables and their distributions, concepts of Compound, truncated and mixture distributions, definition and examples of Singular and Cantor distributions.

### Module II

Discrete distributions- Degenerate, binomial, Poisson, negative binomial, geometric, uniform, hyper geometric distributions, Power series distributions- generalized power series and modified power series distribution, multinomial distribution.

### Module III

Continuous distributions- uniform, normal, exponential, double exponential, beta, gamma, Cauchy, Weibull, Pareto, log-normal and logistic distributions. Pearson system of distributions, bivariate normal and Gumbel's bivariate exponential distribution.

### Module IV

Order Statistics- definition and basic distribution theory of order statistics - joint, marginal and conditional distributions of order statistics of a random sample arising from continuous distributions, distribution of sample median and range.

Sampling distributions- distribution of the mean and variance of a random sample from normal population, Chi-square, t, and F distributions (both central and Non-central), their properties and applications.

**Text books:**

1. Rohatgi, V. K. and Saleh, (2003). An Introduction to Probability Theory and Mathematical Statistics, John Wiley and Sons.
2. Mukhopadhyay, P. (2006). Mathematical Statistics, Books and Allied (P) Ltd., Kolkatta.
3. Gumbel, E.J. (1960). Bivariate exponential distribution, JASA, Vol. 55,pp.698-707.

**References:**

1. Hogg, R.V. and Craig, A.T. (1995). Introduction to Mathematical Statistics, Fifth Edition, Prentice Hall N J
2. Johnson, N. L., Kotz, S. and Balakrishnan, N. (2004). Continuous Univariate Distributions, Vol I Second Edition, John Wiley and Sons ( Asia), PVT Ltd, Singapore.
3. Johnson, N. L., Kotz, S. and Balakrishnan, N. (2004). Continuous Univariate Distributions, Vol II Second Edition, John Wiley and Sons ( Asia), PVT Ltd, Singapore.

**COURSE CODE : ST 223**

**COURSE TITLE : APPLIED STATISTICS AND NUMERICAL METHODS**

**COURSE OUTCOMES**

On completion of the course, students should be able to:

- CO1. Identify the various index numbers and compute them for data sets
- CO2. Explain the concepts of base shifting, slicing and deflating of index numbers
- CO3. Illustrate the applications of index numbers.
- CO4. Understand the need, use, relevance and limitations of official statistics
- CO5. Explain the roles and responsibilities of various organisations like NSSO, CSO etc.
- CO6. Explain the methods of data collection and dissemination in population census

CO7. Identify the roles of various organisations involved in the census procedure.

CO8. Explain the concepts of various measures of fertility and mortality.

CO9. Compute these fertility and mortality measures.

CO10. Distinguish between stationary and stable population.

CO11. Construct abridged life tables.

CO12. Understand the concepts of iteration and interpolation.

CO13. Apply various methods of interpolation to solve numerical problems.

CO14. Solve differential equations using Picard, Euler, Modified Euler and Runge-Kutta methods

### MODULE OUTCOME

Sl. No:	Outcomes	Taxonomy Level
	On completion of each module, students should be able to:	
Module I	M01. Explain the concept of index numbers and understand their applications M02. Explain and exemplify the various methods of constructing index numbers. M03. Distinguish between various index numbers and evaluate their values. M04. Explain the concepts of base shifting, splicing, and deflating. M01. Explain the need, uses, relevance and limitations of 'Official Statistics'. M02. Explain the roles and responsibilities of NSSO, CSO etc. M03. Articulate/exemplify various concepts associated with Population Census.	Understand Apply Evaluate Apply Understand Understand Apply
Module II	M01. Distinguish between the various measures of Fertility and Mortality and evaluate their values. M02. Explain concepts of stationary and stable population, central mortality, force of mortality. M03. Construct abridged life tables.	Evaluate Understand Create
Module III	M01. Articulate the various methods of iteration and interpolation. M02. Carry out various interpolation techniques to solve	Understand Apply



	algebraic equations	
Module IV	M01. Solve differential equations using Picard, Euler, Modified Euler and Runge-Kutta methods.	Evaluate

### **Module I**

Index numbers-definition and application of index numbers, price and quantity relatives, link and chain relatives. Computation of index numbers. Use of averages, simple aggregate, and weighted average methods, Laspeyre's, Paache's, Marshall-Edgeworth, Kelly's, Dorbish-Bowley and Fisher's index numbers. Criteria of a good index number. Consumer price index number. Base shifting, splicing and deflating index numbers.

Official Statistics: Indian statistical system - NSSO, CSO, RGI and their roles and responsibilities

### **Module II**

Vital Statistics- Measurement of Fertility: Crude birth rate, General fertility rate, Age specific birth rate, Total fertility rate, Gross reproduction rate, Net reproduction rate. Measurement of Mortality: Crude death rate, Standardized death rates, Age-specific death rates, Infant Mortality rate, Death rate by cause. Mortality table, stationary and stable population, central mortality rate, force of mortality, construction of abridged life tables.

### **Module III**

Numerical solutions of algebraic equations- method of iteration and Newton Raphson Method, Finite differences, Lagrange, Hermite and Spline interpolation,

### **Module IV**

Numerical differentiation and integration, Numerical solutions of differential equations using Picard, Euler, Modified Euler and Runge-Kutta methods.

### **Text Books:**

1. Mukhopadhyay, P. (1999), Applied Statistics, New Central Book Agency Pvt. Ltd, Calcutta.
2. Goon A.M., Gupta.M.K., and Dasgupta, B. (1986). Fundamentals of Statistics, Vol 2, World Press, Calcutta
3. Sastry, S. S. (2006). Introductory methods of Numerical Analysis (Fourth Edition), Printice Hall, New Delhi.

4. Barlow.R.E and Proschan, F (1985). Statistical theory of reliability and life testing, Holt, Rinehart and Winston.
5. Lawless J.F. (1982), Statistical Models and Methods of lifetime data, John Wiley & Sons, New York.
6. Montgomery, D.C. (2012). Introduction to Statistical Quality Control, 7th edn, John Wiley & Sons, New York.

**References:**

1. Anderson, T.W. (1971). Statistical Analysis of Time Series, John Wiley & Sons, New York.
2. Chatfield, C. (1980). The Analysis of Time Series-An introduction, 2nd edn, Chapman and Hall.
3. Medhi, J. (1992). Statistical Methods-An Introductory Text, New Age, Delhi.
4. Gupta, S.C. and Kapoor, V.K. (2007). Fundamentals of Applied Statistics, Sultan Chand & Sons
5. Grant. E. L. and Leavenworth, R.S. (1996). Statistical Quality Control, 7th edn, McGraw Hill Education (India) Private Limited, New Delhi.

**COURSE CODE : ST 224**

**COURSE TITLE : STATISTICAL QUALITY CONTROL AND RELIABILITY MODELING**

**COURSE OUTCOMES**

On completion of the course, students should be able to:

CO1: Understand the concept of quality and statistical process control.

CO2: Evaluate the performance of a Statistical quality process control

CO3: Perceive the notion of statistical quality product control

CO4: Understand the basic reliability functions and their estimation and utilization to lifetime probability distributions useful in reliability analysis.

CO5: To know the notion of Ageing and interpret the reliability concepts in discrete set up

CO6: Understand the concept of reliability modeling.

## MODULE OUTCOMES

Sl. No	Outcomes On completion of each module, students should be able to	Taxonomy Level
Module I	M01. Explain statistical process control M02. Describe various control charts	Understand Evaluate
Module II	M01. Describe Acceptance sampling inspection techniques M02. Explains Standardization and six-sigma concepts M03. Distinguish chain sampling and continuous sampling	Evaluate Understand Understand
Module III	M01. Explain basic concepts of reliability. M02. Describe reliability of a coherent system. M03. Describe functions of reliability .	Understand Remember Remember
Module IV	MO1. Describe various censoring schemes M02. Describe estimation of parameters based on censored sampling MO3. Nonparametric evaluation of survival function	Understand Apply Evaluation

## COURSE CONTENT

### Module I

Meaning of quality and quality improvement. Basic SQC terminologies-Control limits, Specification limits and Natural tolerance limits. Statistical Process Control-chance and assignable causes of variation, Statistical basis of control charts, – Shewart control charts for variables- ,Mean, Range and S charts. Control charts for attributes - chart for fraction nonconforming- p-chart, np chart ,c and u charts – OC and ARL curve . CUSUM chart.

### Module II

Acceptance sampling by attributes- Single sampling plan, Consumer's and Producer's risk, AQL and LTPD. Type A and Type B OC curves , Rectifying Inspection plan – AOQ , AOQL and ATI curves . Double, Multiple and Sequential sampling plans. Chain and Continuous sampling. Military Standard 105 E and six sigma concepts.

### **Module III**

Basic Reliability concepts - system reliability, series and parallel systems, k out of n systems and its reliability, coherent systems, reliability of coherent systems, cuts and paths. Survival function, hazard function, mean residual life function, life time distributions- exponential, Weibull, gamma distributions-their survival and hazard functions.

### **Module IV**

Observation schemes: censoring and truncation. Concept of left, right and interval censoring. Non-parametric estimation of survival function function: Kaplan-Meier estimator (definition and derivation only) .

### **Text Books:**

1. Montgomery, D.C. (2012). Introduction to Statistical Quality Control, 7<sup>th</sup> edn, John Wiley & Sons, New York.
2. Grant. E. L. and Leavenworth, R.S. (1996). Statistical Quality Control, 7<sup>th</sup> edn, McGraw Hill Education (India) Private Limited, New Delhi
3. Lawless J.F. (1982), Statistical Models and Methods of lifetime data, John Wiley & Sons, New York
4. Barlow.R.E and Proschan, F (1985). Statistical theory of reliability and life testing, Holt, Rinehart and Winston

### **References:**

5. Nelson, W. (1982): Applied life data analysis, Wiley.
6. Cox, D.R. and Oakes, D. (1984): Analysis of Survival Data, Chapman Hall .
7. Duncan A.J.(1959): Quality Control and Industrial Statistics, Irwin, Homewood I
8. Sinha, S. K. (1986) Reliability and Life Testing, Wiley.

**COURSE CODE : ST 225**  
**COURSE TITLE : PRACTICAL I USING R**

**MODULE I**

DIAGRAMMATIC AND GRAPHICAL REPRESENTATION OF DATA: Graphics with R – standard plot functions and its arguments; Bar diagram, Sub-divided bar diagram, Multiple bar diagram, Pie chart, Line chart, Strip chart, Scatter plot, histogram, Ogives, line graphs, Box-and-Whiskers plot, Normality plots-P-P plots, QQ plots.

**MODULE II**

TABULAR REPRESENTATION OF DATA AND STATISTICAL MEASURES: Frequency tables, Contingency tables, Measures of Central Tendency, Measures of Dispersion, Skewness, Kurtosis, Correlation coefficient.

**MODULE III**

TESTS OF HYPOTHESES: Parametric tests: One sample and two samples tests for mean-Z-tests, t-tests; Tests of proportion, Tests of variances- Chi square and F –test; Analysis of variance- One-way and Two way; Non-parametric tests- Chi square tests for proportions, Chi square test for association, Run test, Kolmogorov- Smirnov test of normality, Mann-Whitney U test for equality of means or medians of two independent samples, Wilcoxon test for paired samples, Kruskal-Wallis H test for equality of means of independent samples, Friedman test for more than two dependent samples.

**Examination:** Three Data sets should be given for Data Analysis which cover the topics in three Modules( Should contain a minimum of three objectives) in the syllabus and two data sets should be answered.