UNIVERSITY OF KERALA

Model Question Paper- M. Sc. Examination

Branch : Mathematics

MM211 LINEAR ALGEBRA

(2020 Admission onwards)

Time: 3 hours Max. Marks:75

Part A

Answer any 5 questions from among the questions 1 to 8

Each question carries 3 marks

1. Determine whether $\{(x_1, x_2, x_3) \in F^3 : x_1x_2x_3 = 0\}$ is a subspace of F^3.

2. Prove that any two bases of a finite dimensional vector space have same length.

3. If V and W are finite dimensional vector spaces such that $dimV > dimW$, then prove that no linear map from V to W is injective.

4. Suppose T is a linear map from V to F. Prove that $u \in V$ is not in $null\,T$, then $V = null\,T \oplus \{au : a \in F\}$.

5. Suppose $S, T \in L(V)$ are such that $ST = TS$. Prove that $null\,(T - \lambda I)$ is invariant under S for every $\lambda \in F$.

6. Suppose $T \in L(V)$ and $(v_1, v_2, \cdots, v_n)$ is a basis of V. Prove that if $Tv_k \in span(v_1, v_2, \cdots, v_n)$ for each $k = 1, 2, \cdots, n$, then $span(v_1, v_2, \cdots, v_n)$ is invariant under T for each $k = 1, 2, \cdots, n$.

7. Suppose $T \in L(V)$ and λ is an eigen value of T. Then prove that the set of generalized eigen vectors of T corresponding to λ equals $null\,(T - \lambda I)^{dim\,V}$.

8. Prove that if A and B are square invertible matrices of same size and $AB = I$, then prove that $BA = I$. . $5 \times 3 = 15$

Part B

Answer all questions from 9 to 13

Each question carries 12 marks

9. A. (a) Suppose that U and W are subspaces of V. Prove that $V = U \oplus W$ if and only if $V = U + W$ and $U \cap W = \{0\}$.

   (b) State and prove Linear Dependence Lemma.

OR

B. (a) If U_1 and U_2 are subspaces of a finite dimensional vector space V, then prove that $dim(U_1 + U_2) = dimU_1 + dimU_2 - dim(U_1 \cap U_2)$.

(b) Define basis of a vector space. Prove that every spanning list in a vector space can be reduced to a basis.

10. A. (a) Define null space $null\,T$ of a linear map $T : V \to W$. Prove that $null\,T$ is a subspace of V. Also prove that T injective if and only if $null\,T = \{0\}$.

(b) Define matrix of vector v in a vector space V. Suppose $T \in L(V, W)$ and $(v_1, v_2, \cdots, v_n)$ is a basis of V and $(w_1, w_2, \cdots, w_m)$ is a basis of W. Prove that $\mathcal{M}(Tv) = \mathcal{M}(T)\mathcal{M}(v)$ for every $v \in V$.

OR

B. (a) Prove that a linear map is invertible if and only if it is injective and surjective.

(b) Prove that two finite dimensional vector spaces are isomorphic if and only if they have the same dimension.

11. A. (a) Prove that every operator on a finite dimensional, nonzero complex vector space has an eigen value.

(b) Suppose V is complex vector space and $T \in L(V)$. Prove that T has an upper triangular matrix with respect to some basis of V.

OR

B. (a) If $T \in L(V)$ has $dimV$ distinct eigen values, then prove that T has a diagonal matrix with respect to some basis of V.

(b) Prove that every operator on a finite dimensional, nonzero real vector space has an invariant subspace of dimension 1 or 2.

12. A (a) State and prove Cayley Hamilton Theorem.

(b) Suppose V is a complex vector space. If $T \in L(V)$ is invertible, prove that T has a square root.

OR

B (a) Define minimal polynomial of $T \in L(V)$. Prove that the roots of the minimal polynomial of T are precisely the eigen values of T.

(b) Suppose V is a complex vector space. If $T \in L(V)$, prove that there is a basis of V that is a Jordan basis for T.

13. A. (a) If A and B are square matrices of same size, prove that $trace(AB) = trace(BA)$.

(b) Define determinant of a matrix. Prove that an operator is invertible if and only of determinant is nonzero.

OR

B. (a) Suppose $T \in L(V)$. Prove that the charactrestic polynomial of T equals $det(zI - T)$.

(b) If A and B are square matrices of same size, prove that $det(AB) = det(BA) = (detA)(detB)$.

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$5 \times 12 = 60$