PART A
Answer all questions. Each question carries 4 marks

1. Prove that an analytical function with a constant argument is constant
2. Define critical point and invariant point of a transformation. Find the critical points and invariant points of $W = \frac{1}{z}$
3. Evaluate $\int_C |z| = 1$, where $c$ is the last half of the unit circle $|z| = 1$ from $z = -i$ to $z = +i$
4. Using Newton Raphson method, find the real root of $z = +i$ from $z = -i$ to $1.2$, correct to 3 decimal places. Given that it is near to 2.
5. Using Taylor’s series method, obtain the solution of $\frac{dy}{dx} = 3x + y^2$. Given that $y(0) = 1$. Also find the value of $y$ for $x = 0.1$, correct to 4 places of decimals.

PART B
Answer 1 full question from each module. Each question carries 20 marks.

Module I

6. A) show that the function $f(z) = \frac{x^3y(x+y)}{x+y}$ for $z \neq 0$, even though Cauchy – Riemann equations are satisfied at $z = 0$
   B) Find the analytic function $f(z) = u+iv$, where $u + v = \frac{x}{x^2+y^2}$
   C) Discuss the transformation $W = e^z$
7. A) If $f(z)$ is an analytic function of $z$, prove that $(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})|f(z)|^2 = 4|f'(z)|^2$
   B) Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find its harmonic conjugate and hence find the corresponding analytic $f(z) = u+iv$
   C) Find the bilinear transformation which maps the points $z = 1,i,-1$ into the points $w = i,0,-i$ respectively. Also find the image of $|z| < 1$, under this transformation.

Module II

8. A) Using Cauchy’s integral formula evaluate $\int_C \frac{\cos^2 z + \sin^2 z}{(z-1)(z-2)} \, dz$ where $c$ is the circle $|z| = 3$
   B) Obtain the Laurent’s series expansion of $\frac{z-1}{z^2} \frac{1}{(a+z)}$ in the region $1 < |z+1| < 3$
   C) Evaluate $\int_0^{\infty} \frac{1}{(x^2+1)^2} \, dx$
9. A) Find the nature and location of singularities of the function i) \( f(z) = \frac{1}{z} \)

ii) \( f(z) = z^3 e^z \)

B) Show that \( \int_0^{2\pi} \cos^2 \theta \, d\theta = \frac{\pi}{2} \)

C) Show that \( \int_0^\infty x \sin x \, dx = \frac{\pi}{2} e^{-a} \)

Module III

10. A) Find the real root of the equation \( x^3 - 4x - 9 = 0 \) which lies between 2 and 3, by Regula falsi method

B) Solve by Gauss-Seidel iteration method

\[
\begin{align*}
10x - 2y + z &= 12; \\
x + 9y - z &= 10; \\
2x - y + 11z &= 20
\end{align*}
\]

C) Use Lagrange’s interpolation formula to find \( f(1) \) from the following data

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-8</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

11. A) Using bisection method, find the root of the equation \( e^x - x = 2 \) lying between 1 and 1.4 correct to five places of decimals

B) The population of a town is given below

|------| 20 24 29 36 46 51 |

Population in lakhs

Estimate the population increase during 1946 to 1976 by Newton’s interpolation formula

C) Solve by Gauss elimination method

\[
\begin{align*}
x - y + 5z - 3w &= 5; \\
x + 3y - z - 11w &= 1; \\
5x - 2y + 5z - 4w &= 5; \\
x + 3y - 7z + 2w &= -7
\end{align*}
\]

Module IV

12. A) Find the value of \( \int_0^{1/2} \frac{x}{\sqrt{1-x^2}} \, dx \) using

i) Trapezoidal rule

ii) Simpson’s rule, by taking \( h = \frac{1}{12} \)

B) Apply Runge-Kutta method to find an approximate value of \( y \) when \( x = 0.7 \); Given that \( \frac{dy}{dx} = y - x^2 \) and \( y(0.6) = 1.7379 \)

C) Using Euler’s method find an approximate value of \( y \) corresponding to \( x = 1 \), Given that \( \frac{dy}{dx} = x + y \) and \( y(0) = 1. \) Take \( h = 0.25 \)

13. A) Solve Laplace’s equation \( u_{xx} + u_{yy} = 0 \) satisfying the following conditions

\[
\begin{align*}
u(0,y) &= 0; \\u(3,y) &= 8 + 2y, \quad \text{for } 0 \leq y \leq 3 \\
u(x,0) &= x^2, \\u(x,3) &= 3 x^2, \quad \text{for } 0 \leq x \leq 3
\end{align*}
\]
B) Using modified Euler’s method obtain the values of \( y(0.1) \) and \( y(0.2) \) for the equation \( \frac{dy}{dx} = \frac{x-y}{y+x} \), given that \( y = 1 \) when \( x = 0 \).