FOURTH SEMESTER B.TECH DEGREE EXAMINATIONS
(2013 Scheme)
13.401 ENGINEERING MATHEMATICS III (S)
MODEL QUESTION PAPER

Time: 3 Hours  Maximum Marks: 100

PART –A

Answer all questions. Each question carries 4 marks

1. If u and v are harmonic functions, prove that \((u_y - v_x) + i(u_x + v_y)\) is analytic.
2. Expand \(\frac{1}{z^2}\) as Taylor series about \(z = 2\).
3. Find the poles and residues of \(\frac{1}{z(z-1)}\).
4. By means of Lagrange’s formula prove that \(y_0 = 0.05(y_0 + y_6) - 0.3(y_1 + y_5) + 0.75(y_2 + y_4)\).
5. Find the approximate value of \(\int_0^1 \frac{1}{1+x^2} \, dx\) by Trapezoidal rule.

PART –B

Answer one full question from each module. Each question carries 20 marks

MODULE I

6. a) Show that \(f(z) = \sqrt{|xy|}\) satisfies CR equations at \(z=0\) but not differentiable at \(z=0\).
   b) Find the analytic function \(f(z) = u + iv\) if \(u - v = (x - y)(x^2 + 4xy + y^2)\).
   c) Under the transformation \(w = \frac{1}{z}\), find the image of \(|z - 2i| = 2\).

7. a) If \(f(z)\) is a function with continuous second order partial derivatives show that \(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4 \frac{\partial^2 f}{\partial x \partial y}\).
   b) Discuss the transformation \(w = sluz\).
   c) Find the bilinear mapping that maps \(z = i, -i, 1\) in to \(w = 0, 1, \infty\) respectively.
MODULE II

8. a) Find Laurent’s series expansion about \( z = 0 \) for \( \frac{z^3-1}{z^5+3z^3+6} \) in \( 2 < |z| < 3 \)

b) Using Cauchy’s integral formula, evaluate \( \int_{C} \frac{e^z}{(z-2)(z+3)} \, dz \) where \( C \) is \( |z-2| = \frac{1}{2} \)

9. a) State Cauchy’s Residue theorem and hence evaluate \( \int_{C} \frac{dz}{(z^2+4)} \)

where \( C \) is \( |z-i| = 2 \)

b) Evaluate \( \int_{0}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} \, dx \)

MODULE III

10. a) Solve \( x^3 - 9x + 1 = 0 \) for the root lying between 2 and 4 by Regula falsi method

b) Solve by Gauss–Seidel method

\[
10x + y + z = 12, \quad 2x + 10y + z = 13, \quad 2x + 2y + 10z = 14
\]

11. a) Find by Newton–Raphson method, the real root of \( \log_{e}x - \cos x = 0 \).

b) The population of a certain town in India are as follows.

<table>
<thead>
<tr>
<th>Year</th>
<th>1921</th>
<th>1931</th>
<th>1941</th>
<th>1951</th>
<th>1961</th>
<th>1971</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (in lakhs)</td>
<td>12</td>
<td>15</td>
<td>20</td>
<td>27</td>
<td>39</td>
<td>52</td>
</tr>
</tbody>
</table>

Estimate the population in the year 1925 and 1965.

MODULE IV

12. a) Using Taylor series method, find \( y \) when \( x = 1.3 \), given that \( y' = x^2y - 1 \) and \( y = 2 \) when \( x = 1 \)

b) Use modified Euler’s method to find \( y(0.1) \) when \( \frac{dy}{dx} = x^2 + y \) and \( y(0) = 0.94 \)

c) Evaluate \( \int_{0}^{8} \frac{dx}{2x+3} \) by Simpson’s rule, dividing into 10 equal parts

13. a) Find an approximate value of \( y \) when \( x = 0.2 \) using Runge-Kutta method of order four, given that \( \frac{dy}{dx} = \frac{x}{y+x}, \quad y(0) = 1 \)

b) Solve
\[ u_{xx} + u_{yy} = (x^2 + y^2)e^{xy}, \quad 0 < x < 1, \quad 0 < y < 1, \quad u(0, y) = 1, \quad u(1, y) = e^y, 0 \leq y \leq 1, \quad u(x, 0) = 1, \quad u(x, 1) = e^x, 0 \leq x \leq 1, \]

with \( k = \frac{1}{a} \)