Model Question Paper

FOURTH SEMESTER B.TECH. EXAMINATIONS

Engineering mathematics -IV

(COMPLEX ANALYSIS AND LINEAR ALGRBRA)

(Common to **AFRT**)

(**2013 Scheme**)

Time: **3Hrs** **Max. Marks** :**100**

Part A

Answer all questions. Each question carries 4 marks.

1. Define analytic function. State the necessary condition for a function $f(z)$ to be analytic at a point. Use this to check whether $f(z)=\bar{z}$ is not analytic .

2. Determine the region of the w plane in to which the first quadrant of the z plane is mapped by the transformation $w=z^2$.

3. Evaluate $\int_C (z-a)^{-1} dz$, where C is a simple closed curve and the point z=a is (i) inside C (ii) out side C.

4. Let H= $\{(a-3b, b-a, a) : a, b \in R\}$.Show that H is a subspace of R^3

5. Find a unit vector orthogonal to (1,1,1) and (1,2,-3).

Part :B

Answer any one full question from each module. Each question carries 20 marks.

Module- I

6 a. If $f(z)$ is an analytic function with constant modulus , then prove that $f(z)$ is a constant.

b. If $w=\phi+i\psi$ represents the complex potential of an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$ determine the function ϕ.

c. Show that under the transformation $w=\frac{z-i}{z+i}$, the real axis in the z-plane is mapped into the circle $|w|=1$.

7.a. Show that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied at there.

b. Show that the function $u=e^{2x}(x\cos 2y - y\sin 2y)$is harmonic .Find it's harmonic conjugate and hence find the analytic function .

c. Find the bilinear transformation which maps the point -1, i, 1 of the z-plane on to the points 1,i,-1 of the w-plane respectively

Module- II

8 a. Expand $f(z)=\dfrac{1}{(z-1)(z-2)}$ in the region $0<|z-1|<1$.

b. Use Cauchy's integral formula to evaluate $\int_C \dfrac{e^z}{(z+1)^2}dz$ where C is $|z-1|=3$

c. By integrating around a unit circle , evaluate $\int_0^{2\pi} \dfrac{1}{5-4\cos\theta}d\theta$

9 a. Discuss the singularities of (i) $\dfrac{1}{1-e^z}$, (ii) $ze^{\frac{1}{z^2}}$

b. Evaluate $\int_C \dfrac{e^{2z}}{(z+1)^4}dz$ where C is $|z|=2$.

c. Evaluate $\int_{-\infty}^{\infty} \dfrac{x^2}{(x^2+1)(x^2+4)}dx$

Module- III

10 a. Express v=(2,7,-4) in R^3 as a linear combination of the u_1=(1,2,0), u_2=(1,3,2) and u_3=(0,1,3)

b. Find the dimension of the null space and column space of

$$\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

11a. Show that the mapping T: R^3-R^3 is defined by T(x, y ,z)=(x+z , x+y+2z, 2x+y+3z) is linear . Find a basis for the kernel of T.

b. Find a least square solution to the inconsistent system A**X**=**b** where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$ and **b**=$\begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

Module- IV

12 a. Find an orthonormal basis for the subspace spanned by (1,2,1), (1,0,1) and (3,1,0) in R^3

b. Find a maxima or minima of $5x_1^2 + 5x_2^2 - 4x_1x_2$ subject to the constraint $\mathbf{x}^T\mathbf{x}=1$ where

x=$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

13.a. Reduce the quadratic form $q = 8x_1^2 + 3x_2^2 + 3x_3^2 + 12x_1x_2 - 8x_2x_3 + 4x_1x_3$ to canonical form by orthogonal transformation. Examine the definiteness

b. Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$

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